Evergreening

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Abstract

We develop a simple model of concentrated lending where lenders have incentives for evergreening loans by offering better terms to firms that are close to default. We detect such lending behavior using loan-level supervisory data for the United States. Banks that own a larger share of a firm's debt provide distressed firms with relatively more credit at lower interest rates. Building on this empirical validation, we incorporate the theoretical mechanism into a dynamic heterogeneous-firm model to show that evergreening affects aggregate outcomes, resulting in lower interest rates, higher levels of debt, and lower productivity.

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"Owe your banker £1,000 and you are at his mercy; owe him £1 million and the position is reversed." — J. M. Keynes (1945)

1 Introduction

Following the outbreak of COVID-19 in early 2020, firm profits declined sharply, and governments supported businesses by providing them with subsidized credit. At the same time, concerns emerged that banks would "evergreen" loans—the practice of granting further credit to firms close to default to keep such firms alive. Similar to the government credit programs, such lending behavior may stabilize an economy in the short run, preventing bankruptcies and worker layoffs. After the crisis passes, however, it may contribute to less productive firms remaining in business, leading to the creation of "zombie firms" and depressing aggregate productivity and economic growth (Peek and Rosengren, 2005; Caballero, Hoshi and Kashyap, 2008). For the United States, such worries were frequently dismissed on the basis that evergreening is typically associated with economies experiencing depressions with undercapitalized banking systems, such as Japan in the 1990s, and the U.S. was not thought to be in such a position (Gagnon, 2021).

Assessing whether banks evergreen loans requires a general theory that formalizes such lending behavior. In this paper, we illustrate the economic mechanism that results in evergreening using a stylized model of bank lending. Equipped with this basic framework, we address the following questions. First, is evergreening a general feature of financial intermediation instead of being specific to economies that resemble Japan in the 1990s? If so, can we find empirical evidence for such lending distortions even for the U.S. economy over recent years, when banks were operating with relatively high capital ratios? And finally, what are the macroeconomic implications of evergreening for aggregate outcomes?

We begin our analysis by modifying a simple model of bank-firm lending along two realistic dimensions. First, we assume that a bank owns a firm's legacy debt, resulting in losses in case of firm default. Second, we posit that the bank behaves as a Stackelberg leader and internalizes how the offered lending terms influence a firm's decision to default on existing liabilities. The presence of such concentrated lending can reverse typical lending incentives. In contrast to standard intuition, lenders may offer relatively *better* terms to less productive and more indebted firms closer to the default boundary. By providing more attractive conditions on a new loan contract, a bank raises the continuation value of a firm, thereby reducing the likelihood of default and increasing the chance of repayment of existing debt. All else being equal, larger outstanding debt raises the threat of default and improves a borrower's position vis-à-vis its lender, as captured by the Keynes quote above. Within our static framework, firms with "worse" fundamentals—more debt and lower productivity—pay lower interest rates and invest more. Importantly, the proposed mechanism is distinct from well-known corporate finance theories, such as risk-shifting or debt overhang, and does not hinge on information asymmetries, unhealthy lenders, or depressed aggregate conditions.

To assess whether such lending behavior can be found in practice, we turn to the Federal Reserve's Y-14 data set, which provides detailed loan-level information for the United States. We make use of the fact that the data include banks' risk assessments for each borrower, in particular firms' probabilities of default which we use to measure firm financial distress. Using the fixed effects approach by Khwaja and Mian (2008), we show that banks that own a larger share of a firm's debt lend relatively more to distressed firms at lower interest rates. These effects persist at the firm level, affecting total debt and investment. We obtain these results even outside of a recession when banks were relatively well capitalized, and further show that other prominent theories of evergreening or zombie lending based on bank capital positions cannot explain our findings. Thus, we view our mechanism as a general feature of financial intermediation as opposed to being specific to economies that find themselves in a severe recession with an undercapitalized banking system.

Building on this empirical evidence, we embed the theoretical mechanism into a dynamic heterogeneous-firm model based on the one developed by Hopenhayn (1992), augmented with debt, default, and financial frictions as in Hennessy and Whited (2007), Gomes and Schmid (2010), or Clementi and Palazzo (2016). The dynamic model improves on the static one by endogenizing the joint distribution of firm productivity, debt, and capital, and allows us to study the macroeconomic effects of evergreening. Calibrating the model to U.S. data, we show that evergreening arises in equilibrium and affects firm borrowing and investment decisions. On the one hand, evergreening allows lenders to recover their investments more frequently, and these benefits are passed on to borrowers in the form of lower interest rates. As a result, incumbent firms increase their debt and capital by 1 to 3 percent across different model specifications. On the other hand, the firms that are saved and invest more are the ones that are less productive and prevent new firms from entering. In turn, this reduces aggregate total factor productivity (TFP) by around 0.25 percent relative to an economy with dispersed lenders.

The dynamic model delivers additional insights. We decompose measured TFP losses into three components: firm size, average firm productivity, and misallocation. Most of the drop in TFP is due to firm size: firms are relatively larger in an economy with evergreening, which causes productivity losses under decreasing returns-to-scale production technologies. We further find that firms that benefit from subsidized lending tend to be larger, more leveraged, and less productive—all features that the literature typically associates with zombie firms. However, subsidized firms are also riskier and pay higher interest rates than non-subsidized firms, though lower rates relative to a counterfactual economy without evergreening. Given these differences, we compare various classifications of zombie firms against our measure of whether a firm is subsidized. Definitions based on characteristics such as leverage and productivity as in Schivardi, Sette and Tabellini (2022) tend to correlate with our measure. In a final exercise, we replicate the cross-sectional regression estimates based on model simulations and show that the mechanism generates comparable real effects as in the data.

Related Literature. Our paper relates to the literature on evergreening and zombie lending that emerged from Japan's "lost decade," which started with the collapse of stock and real estate markets in the early 1990s. For this period, Peek and Rosengren (2005) provide evidence of evergreening by showing that poorly performing firms typically experienced an increase in their credit. Lending surges were also associated with weakly-capitalized banks or if banks and firms had strong corporate affiliations.[1] Similarly, Caballero, Hoshi and Kashyap (2008) document a rise in the share of zombie firms, which they define as businesses that pay interest rates below comparable prime rates. Consistent with a model of creative destruction, they show that job creation and destruction declined and productiv-

[1]Within the bank, loan officers may engage in evergreening if they face a lower likelihood of being exposed (Hertzberg, Liberti and Paravisini, 2010). Banks also reduce zombie-lending after on-site inspections (Bonfim et al., 2022).

ity growth stalled in industries that experienced an increase in the share of zombie firms. The presence of zombie firms also spilled over to other firms. In industries with a higher share of zombies, healthy firms experienced a fall in their investment and employment, while their productivity relative to zombies increased.

Building on these seminal contributions, several papers have documented similar evidence of evergreening and real economy effects of zombie firms.[2] These studies span several countries with varying economic conditions. Still, they generally share two main findings: that evergreening is more prevalent among weakly capitalized banks during severe recessions and that zombie firms adversely impact healthy firms and impede firm exit and entry, hindering productivity growth within industries (see Acharya et al., 2022, for a recent survey). We contribute to this literature in the following three ways.

First, we provide a novel theory of evergreening that shows that lenders may be incentivized to recoup their investments by keeping less productive firms alive. Thus far, relatively few papers formalize the ideas of evergreening or zombie-lending theoretically, and a common modeling approach is still lacking. Previous theories have relied on information asymmetries (Rajan, 1994; Puri, 1999; Hu and Varas, 2021), on the premise that banks gamble for resurrection (Bruche and Llobet, 2013; Acharya, Lenzu and Wang, 2021), or that banks delay the recognition of loan losses (Begenau et al., 2021). In contrast, our mechanism assumes full information and does not rely on bank regulation, capital-constrained lenders, or depressed aggregate conditions. Thus, it is not specific to economies that resemble Japan in the 1990s—with undercapitalized banks and a deep recession—but rather describes a general feature of financial intermediation.

The mechanism is also different from the classic problem of debt overhang (Myers, 1977), where equity holders are reluctant to invest in profitable investment projects as benefits could be reaped by existing debt holders, hindering further borrowing. In our framework, more indebted firms receive better loan conditions, enabling them to borrow and invest relatively more — the opposite result. Similarly, our mechanism is related to the idea of sequential lending with non-exclusive contracts as in Bizer and DeMarzo (1992). Contrary to sequential banking, where

[2]Among others, examples are Giannetti and Simonov (2013), Storz et al. (2017), McGowan, Andrews and Millot (2018), Acharya et al. (2019), Andrews and Petroulakis (2019), Acharya et al. (2020), Bittner, Fecht and Georg (2021), Schmidt et al. (2020), Acharya et al. (2021), Chari, Jain and Kulkarni (2021), Banerjee and Hofmann (2022), and Artavanis et al. (2022).

firms borrowing from multiple lenders tend to overborrow, dilute the stakes of preexisting lenders, and have higher default probabilities, our model predicts that firms that borrow from a single (concentrated) lender tend to borrow more but face lower probabilities of default, relative to the case where they would be borrowing from multiple (dispersed) lenders.

Nevertheless, our theory shares some similarities with mechanisms that have been proposed in the literature. For example, Bolton et al. (2016) show that relationship lenders can screen out good borrowers and provide them with relatively cheap financing in a crisis. Cetorelli and Strahan (2006) find that less competition among banks is associated with fewer firms that are larger on average, and Giannetti and Saidi (2019) show that a higher indebtedness of banks to specific industries is associated with stronger incentives to provide credit in times of distress. We share with Becker and Ivashina (2022) the observation that zombie lending may not only be due to bank risk-shifting motives but is also determined by costly corporate insolvency, an important assumption of our framework. Using cross-country firm- and loan-level information, Becker and Ivashina (2022) show that weak insolvency regimes give rise to more zombie lending in crisis years. We also have in common with Hu and Varas (2021) the idea that evergreening may not only be present with capital-constrained lenders but also with healthy ones. In their model, a relationship lender may roll over loans even after bad news about a firm arrives, at the prospect that a market-based lender with less information may lend to such a weak firm in the future.

Our second contribution is quantifying the aggregate effects of evergreening with a calibrated heterogeneous-firm model. Few papers have provided similar assessments, and the results hinge on the specifics of the micro-foundations. In Acharya, Lenzu and Wang (2021), excessive forbearance induces low-capitalized banks to risk-shift and lend to less productive firms, depressing overall output. Tracey (2021) considers a setting in which heterogeneous firms have the option to enter a loan forbearance state, which results in a larger number of less productive firms and lower output. In contrast, in our model, firms do not enter explicit restructuring states to be subsidized by the lender. We find that evergreening depresses TFP primarily thanks to an increase in average firm size.[3]

[3]Our findings relate to Gopinath et al. (2017) who show that a decline in interest rates results in lower aggregate TFP in a model calibrated to Southern Europe in the 2000s (see also Gilchrist, Sim

Last, we contribute to the empirical literature with a new identification approach to detect evergreening behavior and by focusing on large U.S. banks at a time when those were relatively well capitalized—in contrast to prior studies that concentrated on distressed European and Japanese institutions.[4] Blattner, Farinha and Rebelo (2023) use Portuguese data to show that low-capitalized banks extended relatively more credit to borrowers with underreported loan losses following an unexpected increase in capital requirements. Schivardi, Sette and Tabellini (2022) find that weakly capitalized banks in Italy issued relatively less credit to healthy firms—but not zombie firms—during the Eurozone crisis. Consistent with our mechanism, Jiménez et al. (2022) find that Spanish firms were more likely to obtain a public guaranteed loan from banks with higher preexisting debt exposure during the COVID-19 crisis.

2 Static Model

In this section, we develop a simple model of bank-firm lending. We begin by presenting the problem of a firm that decides how much to borrow and invest, taking the interest rate on new credit as given. The firm has preexisting liabilities on which it may choose to default. We then compare the equilibrium outcomes of two economies: (i) one with dispersed lending and (ii) one with concentrated lending, where a single lender owns the firm's outstanding debt and internalizes how loan conditions affect the firm's decision to default on its legacy debt.

Environment. There are two periods $t = 0, 1$. There are two types of agents: firms, indexed by their pre-determined states (z, b), where z is productivity and b is legacy debt, and lenders, who are risk-neutral and have deep pockets.

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and Zakrajšek, 2013; Liu, Mian and Sufi, 2022; Asriyan et al., 2021; and Cingano and Hassan, 2022).

[4] We connect to an extensive body of work that measures how bank health affects the allocation of firm credit (Khwaja and Mian, 2008) and firm outcomes (Chodorow-Reich, 2014). Related to our application, Favara, Ivanov and Rezende (2022), and Ma, Paligorova and Peydró (2021) have used the Y-14 data in this context to investigate the effects of bank capitalization and lender expectations.

2.1 Firm Problem

At the beginning of $t = 0$, the firm may choose to default and obtain a zero value. If it remains in business, the firm has a continuation value equal to $V(z, b; Q)$, which is a function of the legacy debt b, productivity z, and the price of new debt Q that is offered by the lender at $t = 0$, and which the firm takes as a given. The firm therefore defaults if and only if $V(z, b; Q) < 0$. For simplicity, we assume that there is no default at $t = 1$. This assumption is relaxed in the dynamic model.

If the firm does not default, it repays its existing liabilities b, borrows Qb', and invests k' at $t = 0$. At $t = 1$, the firm produces according to a decreasing returns-to-scale technology $z(k')^\alpha$, where $\alpha \in (0, 1)$, and repays debt b' borrowed at $t = 0$. Additionally, the firm faces a borrowing constraint at $t = 0$ that takes the form $b' \leq \theta k'$, where $\theta > 0$.[5] The firm's value, conditional on not defaulting, is

$$V(z, b; Q) = \max_{b',k' \geq 0} -b - k' + Qb' + \beta^f [z(k')^\alpha - b'] \quad \text{s.t.} \quad b' \leq \theta k' \quad , \tag{2.1}$$

where β^f is the firm's discount factor.[6] Appendix A.1 describes the solution to the firm's problem. Under the assumption that the constraint is binding (which we later verify), we characterize the firm's optimal default decision in the following proposition.

Proposition 1. *Firm optimal policies and value (k', b', V) are (i) increasing in Q, (ii) increasing in z. Firm value V is decreasing in b. Additionally, there exists a $Q^{\min}(z, b)$ such that the firm defaults if and only if $Q < Q^{\min}(z, b)$. $Q^{\min}(z, b)$ is (i) strictly increasing in b, (ii) strictly decreasing in z, and (iii) satisfies $\lim_{b\to\infty} Q^{\min}(z, b) = \beta^f + 1/\theta$.*

Equipped with the solution to the firm's problem for a given price of debt Q, we now proceed to study two different forms of determining Q and characterize the equilibria that result from each of them.

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[5] Appendix A.2 shows that all our results hold under more general borrowing constraints of the type $b' \leq g(k')$, which nest standard specifications of earnings-based constraints, for example.

[6] We assume that the firm owns no preexisting stock of capital that would allow it to produce at $t = 0$ and faces no costs of issuing equity. This is without loss of generality: preexisting capital and production in the first period are equivalent to rescaling the net liabilities b. Adding a linear equity issuance cost also increases net liabilities in the first period and introduces an additional distortion as the marginal cost of investment rises, but it does not affect our results.

2.2 Dispersed Lending

In the first economy we consider, there is a continuum of lenders willing to lend to the firm. These lenders are risk-neutral, have deep pockets, and discount payoffs with factor $\beta^k > \beta^f$. Since we assume that there is no default at $t = 1$, perfect competition in the lending market requires that the offered contract satisfies

$$Q = \begin{cases} \beta^k & \text{if } \beta^k \geq Q^{\min}(z,b) \\ 0 & \text{otherwise} \end{cases} .$$

In this equilibrium, all non-defaulting firms borrow at the same interest rate, regardless of (z, b), which implies that marginal products of capital (MPK) are equalized. More productive firms invest more and borrow more, but credit quantities and prices are independent of the amount of legacy debt b.

2.3 Concentrated Lending

We now analyze the equilibrium with concentrated lenders. There are two key differences in relation to the dispersed lending economy. First, the lender internalizes how its choice of Q affects the firm's default decision. Second, lending is non-anonymous because the lender owns the preexisting debt b and understands that this debt is lost in the case of default. We use the terms "lender" and "bank" interchangeably. The lender's problem is given by

$$W = \max_{Q \geq \beta^k} \mathbb{I}[V(z,b;Q) \geq 0] \times \left[b - Qb'(z;Q) + \beta^k b'(z;Q) \right] \quad ,$$

where $\mathbb{I}$ is an indicator function denoting no default at $t = 0$. If the firm defaults at $t = 0$, the lender makes zero profits.[7] Otherwise, the lender recovers b, lends Qb', and obtains b' at $t = 1$, discounted at β^k. Finally, the lender's choice of Q is constrained to be above β^k, as we assume that the firm may access a competitive debt market like the one previously described. We can equivalently write the bank's

[7]For simplicity we assume that there is no recovery in case of default. Our results are qualitatively robust to assuming that there is some recovery as long as it is not full, i.e., default is costly for the lender. We relax this assumption in the quantitative dynamic model, where we allow for partial recovery in case of default.

problem as

$$W = \max_{Q \geq \max\{\beta^k, Q^{\min}(z,b)\}} \left[b + b'(z;Q)(\beta^k - Q) \right] \quad .$$

From this formulation and the fact that $\partial b'(z;Q)/\partial Q > 0$, it is evident that the bank's objective function is strictly decreasing in Q (subject to the constraint on the choice of Q). For this reason, it is optimal for the bank to offer the lowest possible Q as long as $W \geq 0$. The following propositions characterize bank optimal lending.

Proposition 2. *Let $Q^{\max}(z,b)$ denote the maximum Q at which the bank is willing to lend. $Q^{\max}(z,b)$ solves the implicit equation $W(z,b;Q^{\max}) = 0$ and satisfies the properties (i) $Q^{\max}(z,b) > \beta^k$ iff $b > 0$, (ii) it is increasing in b, (iii) it is decreasing in z.*

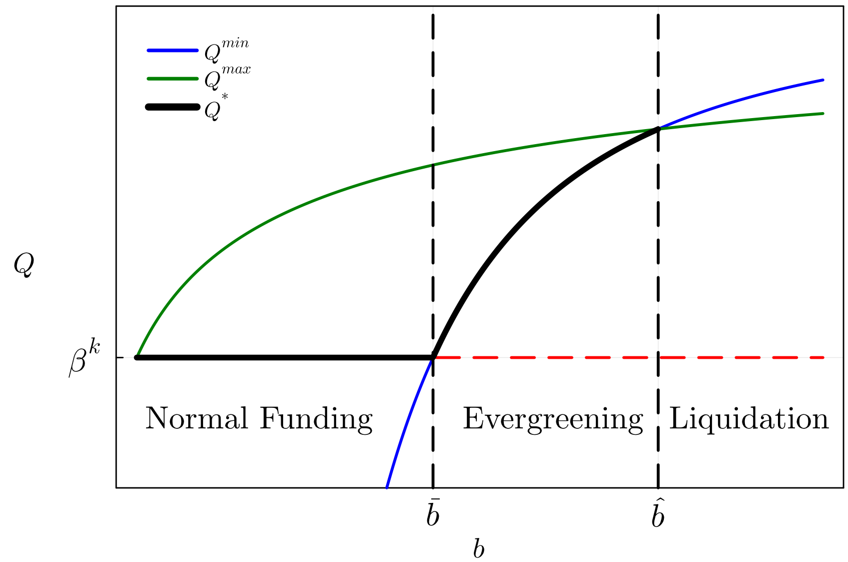
Proposition 3. *The bank's optimal policy can be written as*

$$Q^*(b,z) = \begin{cases} \beta^k & \text{if } Q^{\min}(z,b) \leq \beta^k \leq Q^{\max}(z,b) \\ Q^{\min}(z,b) & \text{if } \beta^k \leq Q^{\min}(z,b) \leq Q^{\max}(z,b) \\ 0 & \text{otherwise .} \end{cases}$$

Let $\bar{b}(z)$ be such that $Q^{\min}(\bar{b}(z),z) = \beta^k$ and $\hat{b}(z)$ such that $Q^{\min}(\hat{b}(z),z) = Q^{\max}(\hat{b}(z),z)$, then (i) $\bar{b}(z) < \hat{b}(z), \forall z$, (ii) $Q^*(b,z)$ is increasing in b, strictly if $b \in [\bar{b}(z), \hat{b}(z)]$, and (iii) $Q^*(b,z)$ is decreasing in z, strictly if $b \in [\bar{b}(z), \hat{b}(z)]$.

Proposition 3 states that, as long as legacy debt is positive, $b > 0$, the bank is willing to offer better terms than those in the competitive market to the firm. Offering more favorable lending conditions allows the bank to recover b by preventing the firm from defaulting. The optimal price of debt Q^* consists of three regions, illustrated in Figure 2.1 for a fixed z. First, as long as $Q^{\min}(z,b) < \beta^k$, the bank can offer $Q^* = \beta^k$ and guarantee that the firm does not default. In this case, the allocation in the concentrated lending economy coincides with the dispersed lending economy ("normal funding"). Second, Proposition 1 states that $Q^{\min}(z,b)$ is increasing in b and decreasing in z. Therefore, for sufficiently high b (or low z), $Q^{\min}(z,b)$ exceeds β^k. In that case, the firm exits in the dispersed lending economy. In the concentrated lending economy, however, and as long as $Q^{\min}(z,b) < Q^{\max}(z,b)$, the bank is willing to keep the firm alive by offering $Q^* = Q^{\min}(z,b) > \beta^k$. These terms are strictly better than those that the firm could ob-

Figure 2.1: Concentrated Lending Economy



Notes: Equilibrium allocation as a function of b, for a given z. The solid blue line is $Q^{\min}(z, b)$, the solid green line is $Q^{\max}(z, b)$, the dashed red line is β^k, and the black line is the optimal policy Q^*.

tain in the competitive market and become more favorable as b increases or z falls. We call this the "evergreening region." In the third region, $Q^{\min}(z, b) > Q^{\max}(z, b)$, and the bank decides to liquidate the firm ("liquidation"). Proposition 1 establishes that the firm's policies are strictly increasing in Q. Thus, equilibrium borrowing and investment follow a similar pattern to that of Q^* in the figure (see Appendix Figure A.1).

Appendix A.4 contains a detailed discussion of how our mechanism relates to and is distinct from existing corporate finance theories, such as the risk-shifting, gambling for resurrection, and debt overhang. It further considers modifications of some of the assumptions of the model, in particular the nature of the contracting protocol and the absence of debt restructuring.

2.4 Discussion

The two-period model isolates the potential advantages and disadvantages of evergreening. On the one hand, evergreening saves firms with too much debt but otherwise viable investment projects that have a positive net present value and generate additional production. On the other hand, less productive firms remain in business and invest more than they otherwise would, potentially absorbing resources that could be better allocated to more productive entrants. However, the static model also leaves several questions unanswered. First and foremost, does the mechanism accurately reflect how banks make lending decisions in practice? We address this question in the next section using detailed loan-level data.

Second, the static model is silent on the macroeconomic consequences of evergreening: it assumes that firms start with certain levels of debt and productivity, but how often do firms end up with states that give rise to evergreening? Do firms potentially acquire more debt today if they know they could be saved tomorrow, a form of moral hazard? Does the survival of such firms prevent the entry of more productive ones? To answer these questions, Section 4 develops a macroeconomic framework that allows for endogenous firm entry and exit, aggregation across firms, and a counterfactual analysis between concentrated and dispersed lending economies.

Firm Distress and Motivation for Empirical Strategy. The static model is simplified to clearly isolate the economic mechanism that generates evergreening in equilibrium. In particular, we abstract from any type of risk, resulting in probabilities of default that take either the value of zero or one depending on firms' initial states. In practice, firms are subject to other types of shocks that affect default beyond indebtedness and productivity. In Appendix A.5, we extend our baseline model to include idiosyncratic firm risk and show that our main qualitative results remain unchanged. Based on a numerical example, Appendix Figure A.3 shows probabilities of default for firms with different levels of legacy debt and the same productivity. The larger b, the higher a firm's probability of default. For low levels of b, firms' default probabilities coincide between the dispersed lending economy and the concentrated lending economy. In contrast, they diverge for intermediate values of b since the single lender offers better credit conditions to the firm, thereby lowering its chance of default. These features motivate our empirical strategy in the following section. We identify distressed firms based on their default probabilities, and require that those are sufficiently elevated, so that their concentrated lenders subsidize them.

3 Empirical Analysis

3.1 Identification Approach

To identify the credit supply effects associated with our theory, we consider a sample of firms that borrow from multiple lenders, which allows us to control for credit demand (Khwaja and Mian, 2008). If anything, this approach makes finding evidence for our theory more challenging since the described mechanism may be stronger if a firm borrows from a single lender instead. Our empirical approach relies on cross-sectional variation in bank exposures to distressed firms. We measure bank exposures according to the share of a firm's debt that banks hold, and classify firms as financially distressed if banks assess their probability of default as elevated. Consistent with our theory, we find that banks that own a larger debt share provide distressed firms with relatively more credit at lower interest rates. We further show that these effects persist at the firm level, affecting total firm debt and investment, and that other prominent theories of evergreening or zombie lending based on bank capital positions cannot explain our findings.

3.2 Data

The main data set of our analysis is the corporate loan schedule H.1 of the Federal Reserve's Y-14Q collection (Y14 for short). These data were introduced as part of the Dodd-Frank Act following the 2007-09 financial crisis. They are typically used for stress-testing and cover large bank holding companies (BHCs).[8] For the BHCs within our sample, the data contain quarterly updates on the universe of loan facilities with commitments in excess of $1 million and include detailed information about the credit arrangements.

Importantly, the data cover banks' risk assessments for each borrower. Among the available assessments, we use the probability of default (PD) in our analysis, which measures the likelihood of a loan nonperforming over the course of the next year. That is, the PD estimates the event that a loan is not repaid in full or that the borrower is late on payments. Banks are supposed to assess the PD at the borrower rather than loan level, which also makes it comparable when multiple banks lend

[8]Until 2019, BHCs with more than $50 billion in assets were required to participate in the collection, and the size threshold was changed to $100 billion subsequently.

to the same firm.[9]

We identify a firm using the Taxpayer Identification Number (TIN). The vast majority of firms within our data are private ones. For these firms, we rely on the banks' own collections of firm balance sheets and income statements that are also part of the Y14 data. To reduce measurement error and to increase the number of observations, we take the median of firm financial variables across all banks and loans for a particular firm-date observation since these data are firm-specific. For the public firms, we instead use information from Compustat on firm financials.

We further apply several sample restrictions. First, we exclude lending to financial and real estate firms. Second, we apply a number of filtering steps that are described in Appendix B, which also includes an overview of the variables that are used. Last, we restrict the sample to 2014:Q4-2019:Q4. The start of the sample is determined by the fact that the risk assessments that we use in our analysis became available at that time. We include information up until 2019:Q4 to ensure that our results are not affected by the COVID-19 crisis. Over this sample, we cover 3,168,276 loan facility observations and 175,406 distinct firms. We identify 2,719 of those firms as public since they can be matched to Compustat. Compared with the years before the 2007-09 financial crisis, banks were relatively well capitalized over our sample period, operating with higher capital ratios and capital buffers as shown in Appendix Figures B.1 and B.2. The U.S. economy was also growing steadily with annual real GDP growth that ranged between 1.4 and 3.8 percent with an average of 2.4 percent. Thus, we intentionally consider a sample of a steady economy with a relatively well capitalized banking system, as our theory does not hinge on poor lender health or depressed aggregate conditions.

3.3 Identifying Credit Supply Effects

In equilibrium, banks and firms may match according to their need and willingness to evergreen loans. To account for such potential links between bank-firm selection and firm credit demand, we follow the fixed effects approach by Khwaja and Mian (2008) to isolate the credit supply effects associated with our mechanism.

[9] See the U.S. implementation of the Basel II Capital Accord for the definition of default (page 69398) and the definition of probability of default (page 69403): https://www.govinfo.gov/content/pkg/FR-2007-12-07/pdf/07-5729.pdf

For firm i and bank j, we estimate regressions of the form

$$\begin{aligned}\frac{L_{i,j,t+2}-L_{i,j,t}}{0.5\cdot(L_{i,j,t+2}+L_{i,j,t})} =& \alpha_{i,t}+\beta_1\text{Debt-Share}_{i,j,t} \\ &+\beta_2\text{Debt-Share}_{i,j,t}\times\text{Distress}_{i,t}+\gamma X_{j,t}+u_{i,j,t} \qquad (3.1)\end{aligned}$$

where $L_{i,j,t}$ is the aggregated amount of credit between a bank and a firm at time t, and the dependent variable measures percentage changes in credit over two quarters. Specifically, we use the symmetric growth rate as an approximation of a percentage change, which allows for possible zero observations at time t and is bounded in the range $[-2,2]$, reducing the potential influence of outliers. We include firm-time fixed effects $\alpha_{i,t}$ into our regressions, which restrict the sample to firms that borrow from multiple banks. The fixed effects control for credit demand if firms have a common demand across their lenders, and we discuss possible violations of this assumption below.

The main regressors are $\text{Debt-Share}_{i,j,t}$, defined as the ratio of outstanding credit $L_{i,j,t}$ between firm i and bank j to total firm debt $Debt_{i,t}$ at time t, and the interaction of this variable with a firm-specific indicator $\text{Distress}_{i,t}$ that equals one if firm i is in financial distress at time t and zero otherwise. We classify a firm as financially distressed if the average PD across all banks that lend to this firm, denoted by $\overline{PD}_{i,t}$, falls within the top decile of the unconditional distribution of $\overline{PD}_{i,t}$ across all firms within our sample. We choose this definition for the following three reasons. First, we use firms' PDs since those can be understood as *sufficient statistics* to measure firm financial distress and because they directly relate to our theory, which is concerned with firms' distance to default. We use a binary indicator variable, as opposed to the continuous variable $\overline{PD}_{i,t}$, since our theory shows that the relation between firm distress and lender's willingness to evergreen loans is inherently nonlinear (as visible in Appendix Figure A.3), and the indicator $\text{Distress}_{i,t}$ is a simple approximation of this relation. Second, we average PDs across banks since banks differ in their assessments, and the average most likely represents a common view.

And third, we consider the top 10 percent as a reasonable cutoff to capture the part of the firm population that has some realistic chance of default. In the data, most firms have PDs that are close to zero as shown in Appendix Table C.1, and even the median firm has a relatively low PD of around 0.8 percent. The cutoff value for the top decile that we use to define a distressed firm is 3.89 percent. Within the top decile, the threat of default is common, with the median firm having a 7.8 percent likelihood of default which rises steeply at the very top of the distribution. As shown below, our results are robust to varying the cutoff value for $\text{Distress}_{i,t}$ around our chosen benchmark, and to excluding firms for which default seems unavoidable (like the ones with high legacy debt in Figures 2.1 and A.3).

We interpret variation in $\text{Debt-Share}_{i,j,t}$ and $\text{Debt-Share}_{i,j,t} \times \text{Distress}_{i,t}$ as capturing credit supply effects, conditional on the fixed effects and other bank-specific controls that are collected in the vector $X_{j,t}$. However, two concerns may invalidate this interpretation. First, nondistressed firms may have a preference for diversifying their borrowing, shifting their demand away from banks at which they have borrowed more in the past, resulting in $\beta_1 < 0$. Second, conditional on finding itself in financial distress, a firm may turn to its concentrated lender from which it has borrowed more in the past, resulting in $\beta_2 > 0$. To exclude such possible demand shifts, we also consider interest rate responses in addition to changes in credit. To this end, we use $r_{i,j,t+2} - r_{i,j,t}$ as a dependent variable in regression (3.1), where $r_{i,j,t}$ denotes the interest rate associated with the credit agreement between firm i and bank j.[10] If our results represent demand shifts, the estimated coefficients from the interest rate regressions should have the same signs as the corresponding ones from the credit regressions. In contrast, if we capture credit supply effects, they should have the opposite sign.

Last, we exclude credit lines and focus on term loans only. That is because credit movements for credit lines largely represent demand changes. Such contracts provide borrowers with the possibility to flexibly draw and repay credit subject to a predetermined limit and at a fixed spread (Greenwald, Krainer and Paul, 2021).[11] As shown below, our key results remain if we include credit lines, but the interest rate responses indicate that the findings may be driven by demand shifts.

[10] In case of multiple contracts for a bank-firm pair, we consider the weighted sum of the various interest rates using the used credit amounts relative to the aggregated credit as weights.

[11] Note that we exclude bank-firm pairs that cover both credit lines and term loans, though a firm may still have a credit line with another bank that is not part of the regression sample or outside of our data.

Table 3.1: Credit Supply to Distressed Firms.

		Δ Credit			Δ Interest Rate	
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt-Share	-21.88**	-17.48**	-22.37***	0.18***	0.11	0.12*
	(8.24)	(8.58)	(7.84)	(0.05)	(0.07)	(0.06)
Debt-Share × Distress	45.60***	38.56***	44.95***	-0.93***	-0.71**	-0.72**
	(9.49)	(10.50)	(12.84)	(0.33)	(0.33)	(0.32)
Fixed Effects						
Firm × Time	✓		✓	✓		✓
Firm × Time × Pur.		✓			✓	
Bank × Time			✓			✓
Bank Controls	✓	✓		✓	✓	
R-squared	0.58	0.6	0.63	0.74	0.74	0.79
Observations	8,647	5,729	8,576	8,407	5,561	8,338
w/ Distress = 1	539	397	531	528	386	520
Number of Firms	887	642	884	867	621	864
Number of Banks	36	34	34	36	34	34

Notes: Estimation results for regression (3.1) multiplied by 100. All specifications include firm-time fixed effects that additionally vary by the loan purpose in columns (ii) and (v). Columns (iii) and (vi) include bank-time fixed effects and the remaining columns include various bank controls: bank size (natural log of assets), return on assets (net income/assets), deposit share (total deposits/assets), loan share (loans/assets), leverage (liabilities/assets), Tier 1 capital buffer (ratio minus requirement), banks' income gap, and the ratio of unused credit lines to assets. Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2014:Q4 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

The estimation results for regressions (3.1) are reported in Table 3.1.[12] Columns (i) and (iv) show our baseline estimates for credit and interest rates. For the credit regressions, we find that $\beta_1 < 0$ and $\beta_2 > 0$, and both are statistically different from zero at standard confidence levels based on two-way clustered standard errors by bank and firm. The negative coefficient associated with Debt-Share$_{i,j,t}$ shows that a nondistressed firm, that has borrowed more from one bank in the past, has relatively less credit growth going forward with that lender. The positive β_1 for the interest rate regressions indicates that these results represent supply effects, possibly indicating that lenders have a preference to reduce their exposure to firms for which they hold a large debt share.

[12] Appendix Table C.2 shows summary statistics for the variables that are part of our main regressions.

The positive β_2 for the credit regression shows that these results change for distressed firms. Relative to a nondistressed firm, one with an elevated risk of default has more credit growth with a bank that holds a larger share of the firm's debt. The negative β_2 for the interest rate regressions shows that these findings represent supply effects. That is, more exposed lenders provide distressed firms with relatively better credit conditions. The results are also quantitatively important. Relative to a nondistressed firm and a hypothetical lender with zero-exposure, a distressed firm has around 46 percent higher credit growth at close to one percentage point lower interest rates with a lender that holds all of a firm's debt.

The remaining columns in Table 3.1 consider alternative specifications of our baseline regression setup. Columns (ii) and (v) extend the firm-time fixed effects by different loan purposes. These regressions are intended to address the possibility that banks specialize in certain types of lending and that firm demand differs across lending types which may be correlated with our regressors of interest (Paravisini, Rappoport and Schnabl, 2021).[13] While the interest rate regressions already provide evidence against such a concern, the estimation results based on the extended fixed effects confirm that our findings reflect supply rather than demand effects. Finally, in columns (iii) and (vi), we include bank-time fixed effects. While the impact of other bank characteristics cannot be estimated separately in this case, our initial findings remain intact with estimated coefficients that are close to our benchmark estimates.

Before continuing, we note two possible reasons why PDs may not be good measures of firm distress. First, banks may misreport these statistics (a concern we address below). Second, they may reflect banks' expectations of future lending decisions. In particular, if a bank intends to save a firm whenever it experiences some distress so that the firm can repay its outstanding debt, the bank may assign a low PD to that firm today. If anything, such risk assessments would downward-bias our estimates in Table 3.1 and therefore make it harder for us to find evidence for our theory. In that sense, our quantitative findings thus far can be viewed as conservative. Taken together, the results provide empirical support for our theoretical mechanism, showing that concentrated lenders—that hold a larger share of

[13] Specifically, we consider the categories "Mergers and Acquisition," "Working Capital (permanent or short-term)," "Real estate investment or acquisition," and "All other purposes" as separate types (see also Appendix Table B.3).

a firm's debt—provide distressed firms with relatively better credit conditions.

In Appendix C, we explore extensions and consider the robustness of our empirical findings along the following dimensions. First, we investigate whether our results can be explained by an alternative channel such as theories of evergreening or zombie lending based on bank capital positions (e.g., gambling for resurrection or risk-shifting) or by a different mechanism of debt forgiveness or restructuring. Second, we test the sensitivity of our findings to the chosen cutoff value for $\overline{PD}_{i,t}$, to the potential misreporting of PDs by banks, to banks being poorly capitalized, and to the disagreement about PDs across banks. Third, we explore alternative regression specifications that extend the firm-time fixed effects by other contract terms, that consider firms that transition into financial distress, and that include credit lines into the analysis. By and large, our findings remain much the same across the various robustness tests and extensions.

3.4 Comparison with Zombie Firm Classifications

Next, we investigate how typical measures of zombie firms from the literature compare with our firm distress indicator and firms' PDs more generally.

To this end, we define zombie firms following the classifications by Caballero, Hoshi and Kashyap (2008), Schivardi, Sette and Tabellini (2022), and Favara, Minoiu and Perez-Orive (2022) since these three measures can be computed based on the available data.[14] In addition, we also define a zombie firm as one that has high leverage and low productivity.[15] This measure is intended to relate to the static model which predicts that such firms experience financial distress. As shown in Appendix Table C.1, the zombie definitions by Schivardi, Sette and Tabellini (2022) and Favara, Minoiu and Perez-Orive (2022), as well as the model-based measure, are positively correlated with $\text{Distress}_{i,t}$. Firms that are considered to be zombies based on these measures also have higher PDs. However, the correlations are not

[14] Caballero, Hoshi and Kashyap (2008) define a zombie firm as one that pays average interest rates on its debt below safe rates. We measure firm-specific safe rates using six-month and two-year government bond rates weighting those by firms' short- and long-term debt ratios. Schivardi, Sette and Tabellini (2022) define a zombie as one with a return on assets below the safe rate (which we approximate using the federal funds rate) and a debt-to-asset ratio above 0.4. Favara, Minoiu and Perez-Orive (2022) define a zombie firm as one with negative sales growth over the previous three years, a leverage ratio above the median across all firms, and an interest coverage ratio below one.

[15] For leverage, we use the 90th percentile across all firms for a particular period as a cutoff. For productivity, we use the 10th percentile of firms' return on assets for a particular period as a cutoff.

perfect and many firms that are considered zombies appear financially sound with PDs close to zero.[16]

Thus, while hard firm characteristics such as leverage and productivity have some predictive power for firms' likelihood of default, many other idiosyncratic reasons also determine financial distress in practice. We therefore view the use of banks' reported PDs—as opposed to some distress definition based on firm characteristics—as the most direct way of relating to our theory, which is concerned with firms' distance to default.

3.5 Firm Level Effects

In a final exercise, we test whether the effects also persist at the firm level, affecting total debt and investment. To this end, we aggregate a firm's borrowing exposures across its lenders, using the debt shares as weights (as in Khwaja and Mian, 2008, for example). This aggregation leads to a regression specification with an intuitive interpretation. For firm i, we estimate

$$\begin{aligned}\frac{y_{i,t+4} - y_{i,t}}{0.5 \cdot (y_{i,t+4} + y_{i,t})} =& \alpha_i + \tau_{m,k,t} + \beta_1 HHI_{i,t} \\ &+ \beta_2 HHI_{i,t} \cdot Distress_{i,t} + \beta_3 Distress_{i,t} + \gamma X_{i,t} + u_{i,t} \ , \end{aligned} \tag{3.2}$$

where $y_{i,t}$ denotes either total firm debt or tangible assets as an approximation for investment, α_i is a firm fixed effect, $\tau_{m,k,t}$ is an industry-state-time fixed effect, and $X_{i,t}$ is a vector of firm controls. $HHI_{i,t}$ are the aggregated debt exposures, defined as $\sum_j (L_{i,j,t}/Debt_{i,t})^2$ which lie in the range $[0, 1]$, and can be interpreted as a Herfindahl-Hirschmann-Index for debt concentration.[17] $Distress_{i,t}$ is the same binary indicator variable that we used above, which equals one if $\overline{PD}_{i,t} \geq 3.89\%$.

We note that, unlike regression (3.1), we are unable to include firm-time fixed effects, as regression (3.2) covers only a single firm observation per period. As a result, the sample now also includes firms with only a single lender. We estimate

[16] The mass of firms with low PDs is relatively large for zombies under all three measures, with more than half of the firms having a PD of 2.8 percent or less. That makes it unlikely that these are all firms that are saved whenever they experience some distress, so that their lenders assign low PDs with such expectations.

[17] Consistent with the previous regressions, we restrict the sample to term loans only. Since we do not cover all firm debt positions, we control for the ratio of observed credit to total firm debt.

Table 3.2: Credit Supply to Distressed Firms - Firm Level.

	Δ Total Debt		Investment		Δ Ave. Interest Rate	
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
HHI	33.71***	32.79***	11.82***	11.81***	-2.21***	-2.22***
	(8.27)	(8.30)	(3.88)	(3.92)	(0.42)	(0.42)
HHI × Distress	13.34***	19.49***	6.88**	7.55**	-0.78	-0.82
	(4.54)	(5.41)	(3.49)	(3.85)	(0.82)	(0.89)
Distress	-4.38***	-7.24***	-2.56***	-2.34***	0.04	-0.01
	(1.38)	(1.83)	(0.71)	(0.86)	(0.10)	(0.13)
Fixed Effects						
Firm	✓	✓	✓	✓	✓	✓
Time × Industry × State	✓	✓	✓	✓	✓	✓
Firm Controls × Distress		✓		✓		✓
Firm Controls	✓	✓	✓	✓	✓	✓
R-squared	0.56	0.56	0.58	0.58	0.44	0.44
Observations	60,636	60,636	71,854	71,854	55,222	55,222
w/ Distress = 1	5,211	5,211	6,195	6,195	4,896	4,896
Number of Firms	14,400	14,400	17,063	17,063	13,021	13,021
Number of Banks	37	37	37	37	37	37

Notes: Estimation results for regression (3.2) multiplied by 100, where $y_{i,t}$ is either total firm debt in columns (i) and (ii) or tangible assets in columns (iii) and (iv). Columns (v) and (vi) estimate regression (3.2) using the change in the average interest rate $r_{i,t+4} - r_{i,t}$ as a dependent variable. All specifications include firm fixed effects, time-industry-state fixed effects where an industry is classified using two-digit NAICS codes, the ratio of observed debt to total debt, and various firm controls: cash holdings, tangible assets, liabilities, debt, net income (all scaled by total assets), and firm size (natural logarithm of total assets). Columns (ii), (iv), and (vi) include interactions of each of the demeaned firm controls with the distress indicator. Standard errors in parentheses are two-way clustered by firm and the bank with the largest debt-share. Sample: 2014:Q4 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

regressions (3.2) at an annual frequency since firm financials are typically updated once per year. The estimation results are shown in Table 3.2.

Columns (i) and (iii) show the baseline results for total debt and investment, respectively, and columns (ii) and (iv) test the robustness of those initial results by including additional interaction terms between the various (demeaned) firm controls and the distress indicator. We find $\beta_1 > 0$ for both outcome variables, which are statistically different from zero at the one percent confidence level. These results can be explained by the fact that—relative to other nondistressed firms—the ones with more concentrated debt positions are potentially younger firms that are growing relatively faster and therefore have a higher demand for additional debt. We further find $\beta_3 < 0$ for both debt and investment, which are again highly statistically significant. That is, relative to other firms with dispersed borrowing, distressed ones have less debt growth and investment over the next year, which can be due to supply restrictions by their lenders.

Interestingly, we find $\beta_2 > 0$ for both debt and investment. Relative to nondistressed firms with dispersed borrowing, a firm that finds itself in distress with concentrated borrowing has relatively more debt growth and investment, in addition to the effect stemming from $\beta_1 + \beta_3 > 0$. The estimates are again strongly statistically significant and quantitatively important, with around 13 percent for debt growth and 7 percent for investment for a firm with a single lender versus a firm with extremely dispersed borrowing ($HHI \approx 0$).

Columns (v) and (vi) repeat the estimation of regression (3.2) using the change in the average interest rate $r_{i,t+4} - r_{i,t}$ that a firm pays on all its debt as a dependent variable. While we do not observe the average interest rate directly, we obtain an approximation using a firm's reported interest expenses divided by its total debt. We note that this imputation likely adds some noise since interest expenses are a flow over a 12-month period in our data, whereas a firm's total debt is a stock at a particular point in time. Nonetheless, we estimate coefficients β_1 and β_2 that are consistent with our previous findings. However, they are also less precisely estimated compared with the regressions for debt and investment, possibly explained by the imputation of the average interest rate.[18]

These findings are consistent with our theoretical mechanism. A concentrated lender subsidizes a firm in financial distress relative to dispersed lenders. Building on this empirical validation of our theory, we embed the mechanism into a dynamic model to study whether such lending behavior can affect aggregate outcomes like capital, productivity, and output.

[18] Alternatively, these results can speak to a mechanism whereby concentrated lenders extend relatively more credit at roughly similar rates to a firm in financial distress compared with dispersed lenders. While our mechanism works via interest rates, it could also occur via credit rationing as in, e.g., Stiglitz and Weiss (1981), with a smaller reduction or no change in rates due to information frictions not present in our model.

4 Dynamic Model

We first present the model setup and the decision problem of a firm. We describe two potential institutional arrangements, as in the static model, that give rise to different debt price functions and therefore to different equilibria. In Section 5, we calibrate the model and compare equilibria under the two arrangements.

4.1 Setup

Environment. Time is discrete and infinite, $t = 0, 1, 2, \ldots$. The economy is populated by a continuum of firms. The distribution of firms is denoted by $\lambda(z, b, k)$, where z denotes productivity, b is debt, and k is capital. Firms endogenously enter and exit the economy, with the mass of entrants denoted by m. For now, we assume that the price of debt is described by some arbitrary function $Q(z, b, k)$ that firms take as given. In the following sections, we present alternative institutional arrangements that provide microfoundations for this function.

Timing. The timing within each period is as follows: (1) firm productivity z is realized, (2) a lending contract Q is offered and depends only on the firm's current state (z, b, k), (3) firms draw additive shocks $(\varepsilon^P, \varepsilon^D)$ to the value of repayment and default, (4) firms decide to default, non-defaulting firms repay their debt, and new firms enter, (5) firms invest, produce, repay, borrow, and pay dividends.

Besides entry, another new feature relative to the static model is the introduction of i.i.d. additive shocks for the firm. This feature is primarily introduced for computational tractability as it smooths the expectation and probability functions for the firm and the lenders (see Dvorkin et al., 2021).

4.2 Firm Problem

Firms have access to a decreasing returns-to-scale production technology with the production function given by $zk^{\alpha}n^{\eta}$, where z is current productivity, k is current capital, and n is labor. The capital share is α, and the labor share is η. The firm hires labor at wage w and invests in new capital k' at a constant unit cost. Capital depreciates at rate δ. Additionally, the firm pays a fixed cost of operation equal to $\mathbf{c}$. The value of repaying conditional on today's state $s = (z, b, k)$ and the offered contract Q is given by

$$V^P(z,b,k;Q) = \max_{b',k',n\geq 0} div - \mathbb{I}[div < 0][e_{con} + e_{slo}|div|] + \beta^f \mathbb{E}_{z'}[\mathcal{V}(z',b',k')|z] \tag{4.1}$$

$$\text{s.t.} \quad div = zk^\alpha n^\eta - wn - k' + (1-\delta)k + Qb' - b - \mathbf{c} \quad , \tag{4.2}$$

$$b' \leq \theta k' \quad . \tag{4.3}$$

The value of repayment is equal to current dividends div plus the continuation value, which is explained below. The firm is also subject to equity issuance costs, with a fixed cost component e_{con} and a linear cost scaled by e_{slo}. Equation (4.2) defines the firm dividend: the value of production, minus the wage bill, minus the new investment net of undepreciated capital, plus new borrowings, minus debt repayments, and minus the fixed cost. Equation (4.3) is a borrowing constraint as in the static model. We refer to the policy functions that solve this problem as $\mathcal{B}(z,b,k;Q)$ and $\mathcal{K}(z,b,k;Q)$, and the optimal labor choice results from a simple static problem.

The firm chooses how much to borrow b' for an offered price of debt Q that is taken as given. In this sort of environment with defaultable debt, a borrowing constraint is required for an equilibrium to be well-defined (see, for example, Ayres et al., 2018). However, due to precautionary behavior arising from the interaction between the expectation of future shocks and equity issuance costs, the borrowing constraint may not necessarily bind.

The firm's value before deciding repayment, after receiving an offer Q, and upon realizing the additive shocks ε^P and ε^D can be written as $V_0(z,b,k,\varepsilon^P,\varepsilon^D;Q) = \max\left\{V^P(z,b,k;Q) + \varepsilon^P, 0 + \varepsilon^D\right\}$, where $V^P(z,b,k;Q)$ is defined in (4.1), and we normalize the value of default to zero.[19] The shocks ε^P and ε^D represent a stochastic outside option for the entrepreneur who runs the firm, and we assume that they follow a type I extreme value distribution (Gumbel), which implies that the difference between the two random variables $\varepsilon = \varepsilon^P - \varepsilon^D$ follows a logistic distribution with scale parameter κ. Given these assumptions, the probability of repayment

[19] Note that we focus on solvency default, not liquidity default as in Ivanov, Pettit and Whited (2021), for example.

today given Q is

$$\mathcal{P}(z,b,k;Q) = \frac{\exp\left[\frac{V^P(z,b,k;Q)}{\kappa}\right]}{1+\exp\left[\frac{V^P(z,b,k;Q)}{\kappa}\right]} \quad . \tag{4.4}$$

We assume that lenders cannot commit to future prices Q. This means that firms take a price function $Q(z,b,k)$ as given in the next period, which allows us to write the expected value of the firm with respect to the shocks $(\varepsilon^{P\prime}, \varepsilon^{D\prime})$ given future states (z', b', k') as

$$\mathcal{V}(z',b',k') = \mathbb{E}_{\varepsilon^{P\prime},\varepsilon^{D\prime}} V_0(z',b',k',\varepsilon^{P\prime},\varepsilon^{D\prime}) = \kappa \log\left\{1+\exp\left[\frac{V^P(z',b',k')}{\kappa}\right]\right\}. \tag{4.5}$$

4.3 Alternative Lending Arrangements

Dispersed Lending Economy (DLE). The first institutional arrangement consists of a purely competitive credit market. It can be thought of as a bond market with a large mass of atomistic lenders. In this case, the price of debt Q is determined by a free-entry condition for lenders. Given $s = (z,b,k)$, we use the notation $Q^c(s)$ to refer to the dispersed lending equilibrium price, which is the price that satisfies the following zero expected-discounted profit condition

$$\begin{aligned} 0 = & -Q^c\mathcal{B}(s;Q^c) + \beta^k \mathbb{E}_{z'}\{\mathcal{P}(z',\mathcal{B}(s;Q^c),\mathcal{K}(s;Q^c))\mathcal{B}(s;Q^c) \\ & + [1-\mathcal{P}(z',\mathcal{B}(s;Q^c),\mathcal{K}(s;Q^c))]\psi(z',\mathcal{B}(s;Q^c),\mathcal{K}(s;Q^c))\} \quad , \end{aligned} \tag{4.6}$$

where ψ is the recovery value in case of default. This value is given by a fraction ψ_1 of the revenue generated by producing one last period and liquidating the undepreciated stock of capital, i.e. $\psi(z,b,k) \equiv \psi_1 \left[\max_n zk^\alpha n^\eta - wn + (1-\delta)k - \mathbf{c}\right]$. The expression for the price resembles the one used in models of sovereign default, with the difference that we have to take into account the firm choices for capital and debt, $\mathcal{K}(s;Q)$ and $\mathcal{B}(s;Q)$, which are determined after Q^c is offered. Note that we assume that lenders have a discount factor larger than that of the firm, $\beta^k > \beta^f$. This assumption ensures that firms never fully escape their constraints, even in the long run (Rampini and Viswanathan, 2013). It is also similar to the existence of a tax advantage of debt as it distorts firms' choice of capital structure towards debt (Kiyotaki and Moore, 1997; Li, Whited and Wu, 2016).

Concentrated Lending Economy (CLE). The second type of credit market we study is one where lenders internalize the firm choices and the possibility of default on current claims b when choosing lending terms. Consequently, such concentrated lenders may offer a different Q that we denote by $Q^r(s)$. The market power that an existing lender can exercise is limited since a large mass of potential lenders stands ready to start a new relationship with a firm.[20] The problem of a lender that has lent b in the previous period to a firm with current capital k and productivity z is

$$
\begin{aligned}
W(s) = \max_{Q^r \geq Q^n(s)} \mathcal{P}(s; Q^r) \Big[b - \mathcal{B}(s; Q^r) Q^r + \beta^k \mathbb{E}_{z'} [W(z', \mathcal{B}(s; Q^r), \mathcal{K}(s; Q^r)) | z] \Big] \\
+ [1 - \mathcal{P}(s; Q^r)] \psi(s) \quad , \qquad (4.7)
\end{aligned}
$$

where $Q^n(s)$ is the price offered by new lenders. Given the free-entry assumption, $Q^n(s)$ is determined by the zero expected-discounted profit condition

$$
-Q^n \mathcal{B}(s; Q^n) + \beta^k \mathbb{E}_{z'} [W(z', \mathcal{B}(s; Q^n), \mathcal{K}(s; Q^n)) | z] = 0 \quad . \qquad (4.8)
$$

Thus, a concentrated lender would like to extract as much surplus as it can, but is constrained by the outside option of the firm to start a new relationship. In addition, the concentrated lender also understands that Q^r affects the probability of survival today $\mathcal{P}(s; Q^r)$ and hence the likelihood of b being repaid. The lender may therefore offer a Q^r that is strictly higher than Q^n. We say that the firm benefits from subsidized lending whenever the prevailing price of debt offered by a concentrated lender is strictly larger than the counterfactual price of debt that the firm would obtain were it to match with a new lender, $Q^r > Q^n$.

4.4 Closing the Economy

New entrants have to pay a fixed cost ω to take a productivity draw $z \sim \Gamma(z)$ and start operating. We assume that new entrants are endowed with a certain amount

[20] In fact, the model is perfectly competitive due to the assumption of free-entry of lenders and no costs of switching lenders. Adding switching costs would generate ex-post market power for lenders. Lender free-entry would still ensure that contracts are ex-ante competitive.

of capital equal to $\underline{k}$. Firms are willing to enter as long as

$$\mathbb{E}_{\Gamma}[\mathcal{V}(z',0,\underline{k})] \geq \omega + \underline{k} \quad . \tag{4.9}$$

Let $\lambda(z,b,k)$ be the measure of firms after entry and exit have taken place. In a stationary equilibrium, the measure λ is the same across periods, and consistent with a law of motion

$$\begin{aligned}\lambda(z',b',k') = &\int_{z,b,k} \Pr(z'|z)\mathbb{I}[\mathcal{B}(z,b,k)=b']\mathbb{I}[\mathcal{K}(z,b,k)=k']\mathcal{P}(z,b,k)\mathrm{d}\lambda(z,b,k) \\ &+\mathrm{m}\int_{z}\Gamma(z)\Pr(z'|z)\mathbb{I}[\mathcal{B}(z,0,\underline{k})=b']\mathbb{I}[\mathcal{K}(z,0,\underline{k})=k']\mathcal{P}(z,0,\underline{k})\mathrm{d}z \ ,\end{aligned} \tag{4.10}$$

where $\mathbb{I}$ is the indicator function, equal to 1 if the condition in brackets is satisfied and 0 otherwise, m is the mass of new entrants, and $\Gamma(z) \equiv \mathcal{U}(z;\underline{z},\tilde{z})$ is the distribution of productivity for entrants, which is a uniform distribution between the minimum value of productivity, $\underline{z}$, which is set at two standard deviations below the mean, and an intermediate value $\tilde{z}$, which is internally calibrated.

With the measure of firms, we can compute labor demand as

$$N^d = \int_{z,b,k} n(z,b,k)\mathrm{d}\lambda(z,b,k) \quad . \tag{4.11}$$

In what follows, we make two alternative assumptions about how to close the economy, which is defined for some function $Q(z,b,k)$ that firms take as given. The key difference between the two equilibrium concepts is whether wages adjust. Under "constant entry," wages do not adjust, and one can therefore interpret the economy as a single industry that is relatively small in terms of the aggregate labor market. Under "constant labor," wages adjust, and the economy rather represents the general equilibrium of an entire economy.

Constant Entry. First, we consider an economy with constant entry by making the assumptions that (i) the measure of entrants is perfectly inelastic, $\mathrm{m} = \bar{\mathrm{m}}$ and (ii) labor supply is perfectly elastic, so it adjusts to be equal to the labor demand as in (4.11). An equilibrium with constant entry is a collection of policy and value functions $(\mathcal{K},\mathcal{B},V^p)$, a constant wage $w = 1$, a measure $\lambda(z,b,k)$, and a constant mass of entrants $\bar{\mathrm{m}}$ such that (a) the policy and value functions solve the firm's problem in (4.1) given the function Q and $w = 1$, (b) a wage $w = 1$ that ensures that the free-entry condition (4.9) is satisfied (possibly with a strict inequality), and (c) the distribution of firms is given by a measure λ that satisfies (4.10).

Constant Labor. Second, we consider an economy with constant labor by assuming that (i) the measure of entrants is perfectly elastic and new firms make zero expected-discounted profits, and (ii) labor supply is constant at $\bar{N}$. An equilibrium with constant labor is a collection of policy and value functions $(\mathcal{K}, \mathcal{B}, V^p)$, an equilibrium wage w, a measure of firms $\lambda(z, b, k)$, and a mass of entrants m such that (a) the policy and value functions solve the firm's problem (4.1) given Q and the wage rate w, (b) a wage rate w that ensures that the free-entry condition (4.9) is satisfied with *equality*, (c) the measure of firms λ satisfies (4.10), and (d) the mass of entrants m is such that the demand for labor (4.11) is equal to $\bar{N}$. This definition resembles the one in Hopenhayn (1992).

5 Quantitative Evaluation

5.1 Calibration

We calibrate the model to an annual frequency, and the parameters we pick are summarized in Table 5.1. We use a combination of external and internal calibration. As our benchmark economy, we choose the model under concentrated lending and the equilibrium with constant labor. Our calibration strategy is based on matching a series of general moments from the literature and the Y-14 data that are typically used as targets in the literature but that are not directly related to the evergreening mechanism. We then show that even our agnostic calibration is able to generate evergreening in equilibrium, and generate patterns that are consistent with the empirical evidence that we document in Section 3, which we take as an external validation of the model.

We pick the entry cost ω such that condition (4.9) is satisfied with *equality* for $w = 1$ and normalize $\bar{N} = 100$. We assume that firm productivity follows an AR(1) process in logs, $\log z' = \mu_z + \rho_z \log z + \sigma_z \epsilon_z$. The associated parameters are taken from Gomes (2001) and Gourio and Miao (2010), with $\mu_z = 0$. The two references report similar values for the persistence of the AR(1) process, which we adopt,

Table 5.1: Model Parameters and Values.

Parameter	Description	Value	Source/Reason
ω	Cost of entry	1.184	Normalize $w = 1$
ρ_z	TFP persistence	0.767	Gomes (2001), Gourio and Miao (2010)
σ_u	TFP volatility	0.110	Gomes (2001), Gourio and Miao (2010)
e_{slope}	Equity issuance cost	0.200	Hennessy and Whited (2007)
δ	Depreciation rate	0.100	Aggregate investment/capital of 10%
α	Production, capital share	0.320	Profit share of 16%
η	Production, labor share	0.480	Profit share of 16%
β^k	Lender discount rate	0.970	Real rate of 3%
ψ_1	Recovery value	0.350	Kermani and Ma (2020)
β^f	Borrower discount factor	0.884	Internally calibrated
$\mathbf{c}$	Fixed cost	0.055	Internally calibrated
κ	Logistic distr., scale	0.225	Internally calibrated
$\tilde{z}$	TFP distr. for entrants	1.147	Internally calibrated
$\underline{k}$	Initial capital	0.805	Internally calibrated
θ	Constraint parameter	1.040	Internally calibrated
e_{con}	Fixed cost of issuing equity	0.010	Internally calibrated

but relatively different values for the standard deviation of the innovations. We choose $\sigma_z = 0.11$, an intermediate value within the range of reported values (0.035 and 0.22). The slope parameter for the linear component of the equity issuance cost is set to a standard value of 0.2, consistent with the estimates in Hennessy and Whited (2007). The depreciation rate is calibrated to $\delta = 0.1$, which is in line with standard values for physical capital depreciation in models calibrated at the annual frequency and helps us match an aggregate investment rate of 10.4%. The production function parameters α and η are set to 0.32 and 0.48, respectively. This is consistent with a capital share equal to 0.4 and a degree of returns to scale of 0.8. This helps us match an aggregate profit share, net of fixed costs, of 17.6%, close to the 16% that we measure in the Y-14 data. The discount factor of lenders is set to target a risk-free rate of around 3 percent, a standard value. The recovery rate is calibrated to $\psi_1 = 0.35$, consistent with the recent evidence in Kermani and Ma (2020). The firm discount factor, the fixed cost of operation, the scale for the logistic distribution, the TFP distribution for entering firms, their initial capital, the collateral constraint parameter, and the cost of issuing equity are internally calibrated and jointly chosen to match a series of moments from the data, presented in Table 5.2.

The model does a relatively good job at matching key moments for the distri-

Table 5.2: Data Moments and Model Fit.

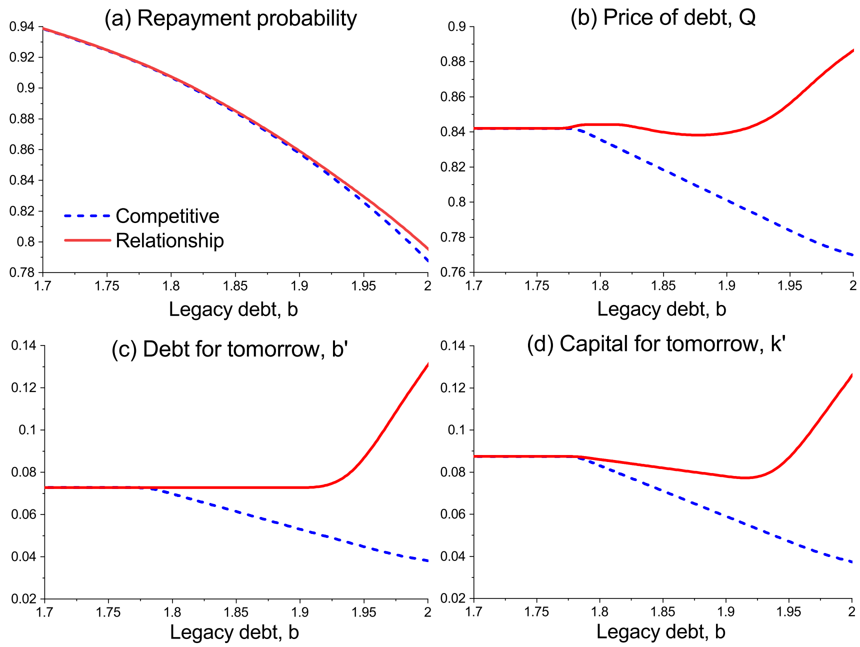
Moment	Source	Data	Model
Market leverage (median)	Y-14/Compustat	0.63/0.57	0.59
Debt over fixed assets (median)	Y-14/Compustat	1.09/1.20	1.04
Investment rate (aggregate)	Y-14/Compustat	0.104/0.14	0.117
Profit share (aggregate)	Y-14	0.16	0.176
Interest rate spread (median)	Y-14	3.46%	4.47%
Exit rate	Hopenhayn, Neira and Singhania (2022)	9.0%	8.8%
Size at entry (relative to mean)	Lee and Mukoyama (2015)	0.60	0.58
Size at exit (relative to mean)	Lee and Mukoyama (2015)	0.49	0.38
TFP at entry (relative to mean)	Lee and Mukoyama (2015)	0.75	0.88
TFP at exit (relative to mean)	Lee and Mukoyama (2015)	0.64	0.86

Notes: Y-14/Compustat moments correspond to unconditional moments between 2014:Q4 and 2019:Q4. Size and productivity at entry and exit are measured in % of average values for incumbent firms, where size is defined as total employment. Investment rate is equal to net investment divided by capital/fixed assets. The profit share is measured as operating profits net of fixed costs divided by output in the model, and as operating surplus divided by sales in the data. The median interest rate spread is computed with respect to a weighted average over contemporary yields on 6-month and 2-year treasury notes, where the weights are given by each firm's short- and long-term debt shares relative to total debt.

bution of firm financials, such as median market leverage and debt relative to fixed assets (capital), as well as the aggregate investment rate. For these moments, we report two numbers, one computed based on firm financials reported in the Y-14 data and another based on Compustat data. Both moments refer to the main sample period of our empirical results, 2014:Q4 through 2019:Q4. The model can generate an exit rate in line with the average value for the last 40 years, as documented by Hopenhayn, Neira and Singhania (2022), as well as a reasonable value for the median interest rate spreads reported on the Y-14 loans.[21] Finally, the model does a relatively good job of matching a series of moments on size and productivity of firms at entry and exit, following Lee and Mukoyama (2015): size is measured as employment, and all of these moments are relative to the unconditional average over the entire distribution. The model can replicate the fact that firms tend to be smaller and less productive than average both at entry and exit.

[21] The median spread from the Y-14 is likely a lower bound as the data covers larger loans of at least $1M (committed amount) issued by relatively large banks.

Figure 5.1: Comparison of Policy Functions.



Notes: Policy functions and values for a firm with the same set of $(z = 0.6, k = 2)$, as a function of b, dispersed lending (blue, dashed) vs. concentrated lending (red, solid) economies.

5.2 Lending Prices and Firm Choices

Figure 5.1 plots policy functions, continuation values, and debt prices for a firm with the same (z, k) in the two economies, as a function of preexisting debt b. We begin by describing the dispersed lending case illustrated by the blue dashed lines, where results are perhaps more standard and intuitive. The firm's value is strictly decreasing in b, which implies the same relation for the probability of repayment (panel a). Similarly, k' is strictly decreasing in b as visible in panel (d). That is because firms with more debt are more likely to realize negative profits, forcing them to issue costly equity. When the marginal value of equity is high, investment is lower, which implies less borrowing due to the borrowing constraint, as shown in panel (c). Finally, panel (b) plots the equilibrium price $Q^c(z, b, k)$. As legacy debt increases, the probability of default in the following period rises, leading to a fall in the dispersed lending price. For high levels of legacy debt, the equilibrium price rises slightly as the firm strongly cuts down on its borrowing but still invests. The red lines correspond to the same policy functions under concentrated lending. For low enough debt, the policies are much the same. However, after a certain point, they begin to diverge. Specifically, panel (b) shows that the price of debt rises earlier with more legacy debt. The higher price of debt reflects the subsidy from the concentrated lender who attempts to prevent firm default. As panels (a), (c), and (d) show, the subsidy affects the probability of repayment, as well as firm choices of capital and debt, which are all larger compared with the dispersed lending case. Note that this plot refers to a particular combination of firm states (k, z) that we keep fixed throughout: in other regions of the state space, the probability of repayment may not be sufficiently sensitive to the price of debt such that the policies look more similar across the two economies.

5.3 Aggregate Effects: Dispersed vs. Concentrated Lending

We assess the impact of introducing concentrated lending in Table 5.3 for the two equilibria mentioned before; one with constant entry and one with constant labor. In each of the columns, we compare moments for the stationary equilibrium under concentrated lending to those same moments for the stationary equilibrium under dispersed lending. The top part of the table corresponds to averages across firms, and the bottom part presents aggregates. By steering a firm's default decision through the offered lending terms, a concentrated lender can recover its previous investment more often, benefiting the lender, all else being equal. However, assuming lenders make zero profits in expectation, incumbent firms reap these benefits by borrowing at lower rates that decrease by 1.24% in the equilibrium with constant entry and by 1.13% with constant labor. The average firm in the CLE is, therefore, more indebted, with market leverage rising by 0.60% with constant entry and 0.54% with constant labor. Firms also become larger by nearly 2.34% with constant entry and 2% with constant labor. The average firm in the CLE is also slightly less productive, and firms exit less often.

Regarding aggregates, both debt and capital increase by over 3% with constant entry and by over 1% with constant labor. The more frequent survival of low-productive firms that invest relatively more impedes the entry of other firms and leads to a shift in the distribution of firm productivity. As a result, measured TFP falls by 0.32% with constant entry and 0.23% with constant labor. While measured TFP is lower, the fact that the CLE uses significantly more capital and labor

Table 5.3: Impact of introducing concentrated lending.

	Δ % with const. entry	Δ % with const. labor
Firm level (Averages)		
Market Leverage	0.60	0.54
Interest rate	-1.24	-1.13
Size	2.34	1.99
Productivity	-0.04	-0.02
Exit rate	-0.70	-0.17
Aggregates		
Debt	3.13	1.04
Capital	3.13	1.04
Labor	2.14	0.00
Output	2.14	0.10
Wage	0.00	0.10
Measured TFP	-0.31	-0.23
Number of firms	0.77	-0.94

Notes: Size is measured in terms of capital. Measured TFP is given by $Y/(K^{\alpha}N^{1-\alpha})$.

results in 2.14% more output with constant entry. However, in the equilibrium with constant labor, total labor is fixed, and output is roughly the same in the two economies (0.10% larger).

Table 5.3 also shows the importance of the market-clearing wage assumption. Under constant entry, the CLE features larger increases in aggregate capital, labor, output, and debt. Note also that the equilibrium concepts differ with respect to the number of firms. With constant entry, the number of firms increases as more firms survive, and the measure of entrants is constant. In contrast, with constant labor, the number of firms declines slightly because firms are larger, which implies that fewer resources are available for new entrants, leading to a drop in firm entry.

5.4 Aggregate Productivity in the CLE and the DLE

Our results suggest that the lending regime affects the average size and profitability of incumbent firms, both of which could affect aggregate productivity. We decompose aggregate productivity under each lending regime into three separate terms: static misallocation in the spirit of Hsieh and Klenow (2009), selection (or dynamic misallocation), and average firm size. First, we explicitly define aggregate output in each economy as $Y = \int_s zk^\alpha n(s)^\eta \mathrm{d}\lambda(s)$. The following result describes the maximum level of output that a planner could achieve by reallocating fixed quantities of factors across a fixed mass of firms.

Proposition 4. *In an economy where a planner can freely reallocate capital and labor across firms to maximize production, for a given mass of firms, aggregate production is given by* $Y^* = M^{1-\alpha-\eta}\mathbb{E}[z^{\frac{1}{1-\alpha-\eta}}]^{1-\alpha-\eta}K^\alpha N^\eta$*, where* $K \equiv \int_s k(s)\mathrm{d}\lambda(s)$*,* $N \equiv \int_s n(s)\mathrm{d}\lambda(s)$ *are the aggregate stocks of capital and labor, respectively. Proof: See Appendix D.1.*

As a direct corollary we can write output in the decentralized economy as

$$Y = \underbrace{\left(\frac{1}{S}\right)^{1-\alpha-\eta}}_{\text{avg. firm size}} \times \overbrace{\mathbb{E}[z^{\frac{1}{1-\alpha-\nu}}]^{1-\alpha-\eta}}^{\text{selection}} \times \underbrace{\frac{Y}{Y^*}}_{\text{static misallocation}} \times \overbrace{K^\alpha N^{1-\alpha}}^{\text{factor qtys.}} \quad ,$$

where $S \equiv N/M$ is the average firm size. The first three terms correspond to measured TFP, $MTFP \equiv Y/K^\alpha N^{1-\alpha}$. MTFP depends on three components: the first term is average firm size. This term appears since firms operate with decreasing returns to scale technology: an economy with more and/or smaller firms has higher MTFP, everything else constant. The second term represents selection, or dynamic misallocation: an economy with more productive incumbents on average has higher MTFP, everything else constant. The final term represents static misallocation in the sense of Hsieh and Klenow (2009). It is equal to 1 in an economy where a constant amount of factor inputs are distributed to equalize marginal products of inputs across firms.

This expression is useful to compare aggregate productivity across different economies: for two economies indexed by i, j, we can decompose the ratio

$$\frac{Y_i}{Y_j} = \left(\frac{1/S_i}{1/S_j}\right)^{1-\alpha-\eta} \times \left(\frac{\mathbb{E}_i[z^{\frac{1}{1-\alpha-\nu}}]}{\mathbb{E}_j[z^{\frac{1}{1-\alpha-\nu}}]}\right)^{1-\alpha-\eta} \times \left(\frac{Y_i/Y_i^*}{Y_j/Y_j^*}\right) \times \left(\frac{K_i^\alpha N_i^{1-\alpha}}{K_j^\alpha N_j^{1-\alpha}}\right) \quad . \tag{5.1}$$

Table 5.4 reports the results of the decomposition of MTFP for the CLE vs. the DLE with constant entry or constant labor. MTFP is lower in the CLE in both cases: the decomposition attributes most of this drop to the size component, as firms are on average larger in the CLE. There is also a small negative contribution from selection, as firm productivity is also lower on average in the CLE. Finally, static

Table 5.4: MTFP decomposition: CLE vs. DLE.

Ratio	% Δ DLE constant entry to CLE	% Δ DLE constant labor to CLE
MTFP	-0.309	-0.227
Size	-0.270	-0.188
Selection	-0.008	-0.004
Static Misallocation	-0.032	-0.035

misallocation is worse in the CLE, suggesting that subsidized lending also worsens static efficiency. However, it accounts for only around 10% of MTFP losses, with the bulk arising from firm size. This suggests that traditional measures of static misallocation, such as the standard deviation of MPK, may not be informative regarding productivity losses generated by lending arrangements.

5.5 Subsidized vs. Non-Subsidized Firms

How do subsidized and non-subsidized firms differ in the CLE? Table 5.5 explores this question, reporting medians for different individual firm characteristics, depending on whether those firms are subsidized. Recall that a firm is subsidized when it pays an interest rate to a concentrated lender that is below the rate that it would pay if it were to match with a new lender (regardless of the rate that a firm with the same states would pay in the dispersed lending economy, where no firms are subsidized). The table shows that subsidized firms are around 130% larger than non-subsidized firms. However, they are also around 8% less productive. Still, the size effect outweighs the lower productivity, and the median subsidized firm has around 46% larger output. Subsidized firms are also more leveraged and pay higher interest rates despite the subsidy, because they are riskier compared with non-subsidized firms.

Note that the subsidized firms have most of the characteristics that the literature typically associates with "zombie firms": they are large, unproductive, indebted, unprofitable, and older. Interestingly, however, and despite the subsidy, these firms pay higher interest rates as they tend to be closer to default (the probability of survival is almost 8 pp lower). This puts in question empirical classifications of zombie firms that are based on costs of borrowing being below market or below average for a given peer group (as in Caballero, Hoshi and Kashyap, 2008). Subsidized firms in our model are ultimately risky firms, and thus they pay rela-

Table 5.5: Subsidized vs. non-subsidized Firms in the CLE (medians).

	Non-subsidized	Subsidized	Δ %
Capital	0.75	1.72	128.5
Productivity	1.02	0.94	-8.0
Output	0.41	0.60	46.1
Payouts/assets	0.05	-0.01	-114.4
Market leverage	0.53	0.80	50.6
Interest rate	7.75	10.02	29.2
Probability of survival	0.96	0.89	-7.6
Interest-coverage ratio	1.67	0.45	-73.1
Age	7.87	10.17	29.2

tively higher interest rates. However, these interest rates are not as high as those offered by a new lender without evergreening incentives—a counterfactual that cannot be observed in the data.

Subsidized vs. Zombie Firms. While there is a large empirical literature that attempts to classify zombie firms, there is no single definition of what constitutes one, and a wide range of classification methodologies have been proposed in the literature. We focus on the measure by Favara, Minoiu and Perez-Orive (2022) (FMP), who quantify the number of zombie firms in the U.S. using a similar dataset to ours. They classify a firm as a zombie if it satisfies the following three conditions: (i) leverage above the median, (ii) an interest coverage ratio below 1, and (iii) average negative sales growth over the past 3 years. Given our calibration, we find that 5.7% of firms satisfy this definition in the stationary equilibrium with concentrated lending. This is consistent with the estimates of FMP, who find a zombie firm share of 5.6%-5.7% between 2017 and 2019. This is a completely untargeted and relevant moment; thus, we take it as a measure of external validation of the model calibration.

Given the differences between subsidized and non-subsidized firms reported in Table 5.5, we also assess how various zombie definitions from the literature correlate with whether a firm receives a subsidy or not in our model (our precise definition of a zombie). Details of this exercise are left to Appendix D.2. We find a high correlation with the measure proposed by Schivardi, Sette and Tabellini (2022), who classify a firm as a zombie if it has (i) a return on assets below the risk-free rate and (ii) leverage above 40%. Overall, classification measures that put more emphasis on profitability and leverage perform better.

5.6 Discussion: Model vs. Data

Our model analysis focuses on two extreme cases: all firms in the economy either borrow from concentrated or from perfectly dispersed lenders. In practice, however, there is substantial variation of lender concentration across firms in the data.

If firms were able to choose the lending regime in our setup, concentrated lending would become dominant. To see this, consider first the case where firms can choose the regime upon entry, and commit to it thereafter. As Table 5.3 shows, the wage rate is higher in the concentrated lending economy vs. the dispersed lending economy, which reflects a higher ex-ante value of entry in the concentrated lending economy for a given wage rate. The higher value of entry is due to differences in lending technology that allow for implicit restructuring in the concentrated lending economy. Second, if we allow firms to choose the regime period-by-period, we find in our calibration that the value function is weakly larger in the concentrated lending economy given a wage rate for any combination of state variables.

The fact that there is variation of lending regimes in the data suggests that other factors govern this choice that are not fully captured by our model; or some firms may not even be able to make this choice at all. To account for such factors, we set up a version of the model where a fraction ϕ of entrant firms is exogenously assigned to dispersed lenders, while all other entrants are exogenously assigned to concentrated lenders. While this version of the model still considers two extreme lending regimes, it generates regime variety in equilibrium by mixing the two.[22] We calibrate ϕ to match the average within-firm HHI of lending in the Y-14 data, the same measure we employ in Section 3.5. Our baseline estimate for this measure is 0.91 which implies $\phi = 0.09$.[23]

[22] For computational tractability, we set up this economy under constant entry, with $w = 1$.

[23] This is the average HHI for firms for which we observe at least 90% of total debt and is computed under the assumption that all unobserved debt is as dispersed as observed debt. This is likely to be a conservative estimate as the Y-14 data is tilted towards larger firms with more dispersed borrowing: average fixed assets of such firms in our sample is \$10.7 M, versus \$3.9 M for the entire economy (aggregate fixed assets from the BEA divided by total number of firms from County Business Patterns). Alternatively, we could pick ϕ to match other aggregate moments. Appendix Figure D.1 shows that the zombie measure by Favara, Minoiu and Perez-Orive (2022) is strictly decreasing in ϕ and targeting it by choosing ϕ would imply an even lower choice of ϕ close to zero to match the share of zombie firms measured in the data.

Based on this calibration, we investigate whether the model generates cross-sectional predictions that are in line with the patterns we find in the data. To this end, we simulate a large number of panels and replicate the empirical specification in Section 3.2 for each. In particular, we regress the (symmetric) growth rates of capital and debt on a measure of lender concentration, a measure of distress, and the interaction between the two. Firms with dispersed borrowing are assigned $HHI_{i,t} = 0$, while firms with concentrated borrowing are assigned $HHI_{i,t} = 1$. We define $Distress_{i,t} = 1$ if a firm borrowing from concentrated lenders receives subsidized credit or if a firm borrowing from dispersed lenders would have been subsidized by concentrated lenders given its current states, and zero otherwise.

The results are reported in Appendix Table D.2, which we compare to the ones in Table 3.2. The model generates the same qualitative predictions we find in the data: more concentrated borrowing is associated with higher capital and debt growth. At the same time, distress is related to lower capital and debt growth, everything else constant.[24] Importantly, the coefficient on the interaction between lender concentration and distress is positive; that is, distressed firms show additional debt and capital growth due to the subsidized borrowing they receive. The coefficient for the investment regression is close to its empirical counterpart, providing evidence that the model mechanism generates comparable real effects. If anything, the model slightly understates the importance of the mechanism. It should be noted that no moment that is directly related to the mechanism is explicitly targeted as part of our calibration strategy, and yet the model produces similar results that we find in Section 3. We also find that these regression results are largely insensitive to the choice of ϕ, which merely changes the fraction of firms with dispersed borrowing.

6 Conclusion

Up to this point, the literature has largely associated zombie lending or evergreening with economies that are in a depression and have severely undercap-

[24] The quantitative results between model and data differ for those coefficients, possibly because firms with concentrated borrowing are younger ones that are still growing in practice leading to a more positive coefficient on $HHI_{i,t}$ in the data. Distressed firms may be slower to adjust their capital and debt in practice as they face capital adjustment costs and various other frictions that are absent from the model, leading to a less negative coefficient on $Distress_{i,t}$ in the data.

italized banks. The main empirical contributions focus on cases that fit these descriptions—Japan in the 1990s and periphery countries during the Eurozone crisis more recently. In this paper, we take a different perspective. We theoretically and empirically argue that evergreening is a general feature of financial intermediation—taking place even outside of depressions and within economies that have well-capitalized banks.

Our proposed theoretical mechanism builds on an intuitive idea. To recover its past investment, a lender has incentives to offer more favorable lending terms to a firm close to default to keep the firm alive. We then explore the empirical relevance and macroeconomic consequences of this general theory of evergreening. We find empirical support for the mechanism in the context of large U.S. banks, at a time when those were thought to be relatively well-capitalized. Using a calibrated dynamic model, we find that evergreening has negative aggregate effects for TFP, mainly due to its role in increasing average firm size. Exploring how similar versions of our proposed mechanism may apply to other settings, such as the mortgage market as in Gupta (2022), is a salient path for future research.

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ONLINE APPENDIX

A Static Model

A.1 Proofs

Proof of Proposition 1 With a binding constraint, the closed-form expressions for the optimal capital stock and the level of new debt are

$$k'(z;Q) = \left(\frac{\beta^f \alpha z}{1-\theta(Q-\beta^f)}\right)^{\frac{1}{1-\alpha}} \quad , \quad b'(z;Q) = \theta k'(z;Q) \quad , \tag{A.1}$$

and the value function can be written in closed-form

$$V(z,b;Q) = -b + \left(\frac{1}{\alpha} - 1\right) \frac{(\beta^f \alpha z)^{\frac{1}{1-\alpha}}}{[1-\theta(Q-\beta^f)]^{\frac{\alpha}{1-\alpha}}} \quad . \tag{A.2}$$

This characterizes the firm's problem for an arbitrary price of debt Q, which is taken as given. We restrict $Q \leq \beta^f + 1/\theta$ to ensure that policy and value functions are well-defined, and later confirm that this restriction is satisfied in equilibrium. Equations A.1 and A.2 show that the firm's policies and value are all strictly increasing in productivity z and the price of debt Q. Additionally, firm value is strictly decreasing in the amount of legacy debt b. Since A.2 is strictly increasing in Q, it follows that there exists a unique $Q^{\min}(z,b)$ such that the firm defaults if and only if $Q < Q^{\min}(z,b)$. A closed-form for this threshold can be found by solving $V(z,b;Q^{\min}) = 0$, and is given by

$$Q^{\min}(z,b) = \beta^f + \frac{1}{\theta} - \frac{(\beta^f \alpha z)^{\frac{1}{\alpha}}}{\theta} \left(\frac{1-\alpha}{\alpha b}\right)^{\frac{1-\alpha}{\alpha}} \quad .$$

From here, the comparative statics follow immediately, $Q^{\min}(z,b)$ is: (i) strictly increasing in b, (ii) strictly decreasing in z, and (iii) converges to $\beta^f + 1/\theta$ from below as $b \to \infty$. This ensures that as long as $Q \leq Q^{\min}$, the firm's policies are always well-defined as long as the constraint is binding.

Proof of Proposition 2 Since the bank's objective is strictly decreasing in Q, the implict equation that defines the maximum Q at which the bank makes non-negative profits is given by

$$W(z,b;Q^{\max}) = 0 \Leftrightarrow b + [\beta^k - Q^{\max}(z,b)]\theta \left(\frac{\beta^f \alpha z}{1-\theta(Q^{\max}(z,b)-\beta^f)} \right)^{\frac{1}{1-\alpha}} = 0\,,$$

First, note that since $b \geq 0$, this equation can only hold with equality if $Q^{\max}(z,b) \geq \beta^k$, strictly if $b > 0$. Second, we can use the implicit function theorem to establish the following results in the proposition:

$$\begin{aligned}
\frac{\partial Q^{\max}(z,b)}{\partial z} &= -\frac{Q-\beta^k}{z\left[1-\alpha+\theta\frac{Q-\beta^k}{1-\theta(Q-\beta^f)}\right]} < 0 \\
\frac{\partial Q^{\max}(z,b)}{\partial b} &= \left\{ \theta \left[\frac{\beta^f \alpha z}{1-\theta(Q-\beta^f)} \right]^{\frac{1}{1-\alpha}} \left[1 + \frac{\theta}{1-\alpha}\frac{Q-\beta^k}{1-\theta(Q-\beta^f)} \right] \right\}^{-1} > 0
\end{aligned}$$

Proof of Proposition 3 Since the bank's profit function is strictly decreasing in Q, the bank will try to offer the lowest possible Q as long as profits are positive, i.e. as long $Q \leq Q^{\max}(z,b)$. If $Q^{\min}(z,b) \leq \beta^k \leq Q^{\max}(z,b)$, then it is profitable to lend but the offered price of debt cannot go below β^k due to the firm's participation constraint. If $\beta^k \leq Q^{\min}(z,b) \leq Q^{\max}(z,b)$, it is profitable to lend and the bank offers $Q^* = Q^{\min}(z,b)$. As soon as $Q^{\min}(z,b)$ exceeds $Q^{\max}(z,b)$, it is no longer profitable to lend and the bank liquidates the firm. We can find the thresholds at which the bank policies change.

Let $\bar{b}(z)$ be the point at which $Q^{\min}(\bar{b}(z,z)) = \beta^k$. This threshold can be found by using the expression for $Q^{\min}$, setting it equal to β^k and solving for b:

$$\bar{b}(z) = \frac{1-\alpha}{\alpha} \left[\frac{\alpha \beta^f z}{(1-\theta(\beta^k-\beta^f))^\alpha} \right]^{\frac{1}{1-\alpha}}$$

Let $\hat{b}(z)$ be the point at which $Q^{\min}(\hat{b}(z),z) = Q^{\max}(\hat{b}(z),z)$. This point can be found by plugging the expression for $Q^{\min}(z,b)$ in the implicit equation that defines $Q^{\max}(z,b)$, and solving for b:

$$\hat{b}(z) = (1-\alpha)\left[\frac{\beta^f z}{(1-\theta(\beta^k-\beta^f))^\alpha}\right]^{\frac{1}{1-\alpha}}$$

It is straightforward to show that $\bar{b}(z) < \hat{b}(z)$ for any z. The properties of $Q^*(z,b)$ follow from those of $Q^{\min}(z,b)$ in the relevant region, when $b \in [\bar{b}(z), \hat{b}(z)]$ for a given z.

A.2 General Form of Borrowing Constraint

In this appendix, we show that several results of the static model hold for the case where the firm faces a general constraint of the type $b' \leq g(k')$, with $g, g' \geq 0$ and $g'' \leq 0$. Note that many types of borrowing constraints, such as no default constraints, are special cases of this general form. With such a general constraint, the choice of capital cannot be solved in closed form, and is implicitly given by

$$\beta^f z\alpha(k')^{\alpha-1} - 1 + (Q-\beta^f)g'(k') = 0 \quad .$$

Note that as long as the constraint binds, all the comparative statics for k' extend to b' due to monotonicity of g, i.e. $\frac{\partial b'(z;Q)}{\partial Q} = g'(k)\frac{\partial k'(z;Q)}{\partial Q}$. We can use the above expression to obtain the implicit derivatives

$$\begin{aligned}
\frac{\partial k'(z;Q)}{\partial Q} &= \frac{g'(k')}{\beta^f z\alpha(1-\alpha)(k')^{\alpha-2} - Qg''(k')} \geq 0 \quad , \\
\frac{\partial k'(z;Q)}{\partial z} &= \frac{\beta^f \alpha(k')^{\alpha-1}}{\beta^f z\alpha(1-\alpha)(k')^{\alpha-2} - Qg''(k')} > 0 \quad .
\end{aligned}$$

It is also straightforward to show that

$$\frac{\partial V(z,b;Q)}{\partial Q} = b' \geq 0, \qquad \frac{\partial V(z,b;Q)}{\partial z} = \beta^f(k')^\alpha \geq 0, \qquad \frac{\partial V(z,b;Q)}{\partial b} = -1 < 0 \quad .$$

The following derivations show that it is still possible to prove Propositions 1-3 for the general borrowing constraint $b' \leq g(k')$.

Proof of Proposition 1. Given that $V(z,b,;Q)$ is increasing in Q, the threshold $Q^{\min}(z,b)$ exists for $b>0$, it is now implicitly defined by

$$0=-b+Q^{\min}b'(z,Q^{\min})-k'(z,Q^{\min})+\beta^{f}[z(k'(z,Q^{\min}))^{\alpha}-b'(z,Q^{\min})]$$

Applying the implicit function theorem allows us to derive the comparative statics

$$\frac{\partial Q^{\min}(z,b)}{\partial z}=-\frac{\beta^{f}(k'(z,Q^{\min}))^{\alpha}}{b'(z,Q^{\min})}<0,\qquad \frac{\partial Q^{\min}(z,b)}{\partial b}=\frac{1}{b'(z,Q^{\min})}>0 \quad .$$

Proof of Proposition 2. $Q^{\max}$ now solves the implicit equation

$$b+[\beta^{k}-Q^{\max}]b'(z;Q^{\max})=0 \quad .$$

Clearly, $Q^{\max}\geq\beta^{k}$ for $b\geq 0$, as $b'(z;Q)\geq 0$. Additionally, applying the implicit function theorem allows us to derive the relationships

$$\frac{\partial Q^{\max}(z,b)}{\partial b}=\frac{1}{b'(z;Q^{\max})+(Q^{\max}-\beta^{k})\frac{\partial b'(z;Q^{\max})}{\partial Q}}>0 \quad ,$$
$$\frac{\partial Q^{\max}(z,b)}{\partial z}=-\frac{(Q^{\max}-\beta^{k})\frac{\partial b'(z;Q^{\max})}{\partial z}}{b'(z;Q^{\max})+(Q^{\max}-\beta^{k})\frac{\partial b'(z;Q^{\max})}{\partial Q}}<0 \quad .$$

Proof of Proposition 3. Proposition 3 follows the same arguments as in the main text. The comparative statics with respect to $Q^{*}(b,z)$ follow from those of $Q^{\min}(z,b)$.

A.3 Parametrization for Numerical Examples and Additional Figures

The static model has four parameters: $\alpha,\beta^{f},\beta^{k},\theta$. All plots are based on the parametrization in Table A.1.

Table A.1: Static Model Parametrization

Parameter	Description	Value
α	Returns to scale	0.35
β^f	Discount factor Firm	0.90
β^k	Discount factor Lender	0.98
θ	Borrowing constraint	0.70

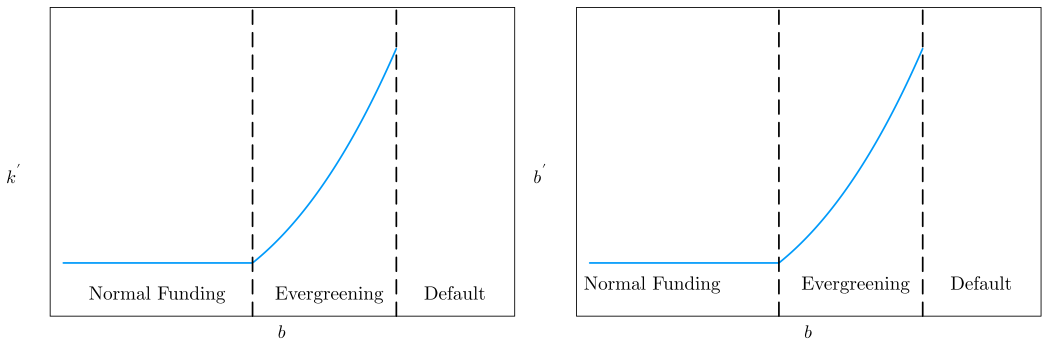


Figure A.1: Equilibrium investment and borrowing policies as a function of b.

A.4 Discussion of the Static Model

A.4.1 Relation to Existing Corporate Finance Theories

Our proposed mechanism is distinct from phenomena such as risk-shifting, gambling for resurrection, or debt overhang. Risk-shifting and gambling for resurrection postulate that distressed borrowers have incentives to invest in risk-increasing negative NPV projects under limited liability (e.g., Jensen and Meckling, 1976). That is because they can reap the benefits if the investments go well, but creditors bear the costs otherwise. Bruche and Llobet (2013) and Acharya, Lenzu and Wang (2021) build on this idea to explain why banks engage in zombie-lending. In contrast, in our framework, banks do not borrow and are therefore not subject to limited liability, firms do not default *following* their investments, and there is no uncertainty, preventing such risk-shifting from occurring.

According to the debt overhang theory, highly indebted borrowers underinvest since the potential profits would primarily accrue to the current creditors, hindering further borrowing (e.g., Myers, 1977). The debt overhang theory relies on the timing that the outstanding (long-term) debt matures *after* the investment decision takes place. In contrast, in our framework, the timing of these decisions is reversed, legacy debt is short-term, and highly indebted firms "overinvest," in the sense that their MPKs are lower than the ones of less indebted firms.

A.4.2 Contracting Protocol

Our benchmark model assumes a specific contracting protocol based on a Stackelberg game. The concentrated lender is the leader (offering Q), and the firm is the follower (choosing b', k' based on Q). One could think of alternative arrangements where the lender sets the price Q and the quantity of debt b' in a take-it-or-leave-it offer. This appendix derives the solution to such a contracting protocol. In this case, the lender implicitly chooses the firm's investment while extracting maximum surplus by setting the firm's value to zero. This solution is equivalent to the lender owning the firm, who undertakes the project without investment distortions. This effectively eliminates agency problems between the firm and its lenders. A consequence of the choice of capital being undistorted is that it no longer depends on the amount of legacy debt b. In contrast to these predictions, we show in Section 3 that evergreening in the data is associated with more favorable lending conditions with respect to credit quantities, interest rates, and investment, with this alternative contracting protocol being unable to generate the latter result. We therefore view our benchmark model as the empirically relevant case, where there is a link between financing conditions and real outcomes.

Our benchmark model is a Stackelberg game where the lender offers Q and the firm chooses how much to borrow for a given Q. Here, we consider an alternative case where the concentrated lender offers a contract that specifies both an interest rate Q and a repayment amount b'. We focus on the more interesting case where $\beta^k < Q^{\min}(z, b)$, so that the firm is in the evergreening region. Thus, the firm can either accept the (Q, b')-offer or default. Taking the firm's decision into account, the concentrated lender is able to extract the maximum surplus from the contract, offering (Q, b') such that $V(z, b; Q) = 0$. This is equivalent to

$$0 = -b + Qb' - k'(z, b; Q, b') + \beta^f \left[zk'(z, b; Q, b')^\alpha - b' \right] \quad ,$$

where $k'(z; b', Q)$ is the firm's optimal choice of capital, given the states (z, b) and the offered contract (Q, b').

First, assume that the firm is unconstrained, i.e. $b' < \theta k'(z, b; Q, b')$. Its capital policy is independent of lending terms and given by $k' = \left(\beta^f z\alpha\right)^{\frac{1}{1-\alpha}}$. The concentrated lender's problem is then

$$\begin{aligned} &\max_{Q,b'} W = b - Qb' + \beta^k b' \\ \text{s.t.} \quad & 0 = -b + (Q-\beta^f)b' + (\beta^f z\alpha)^{\frac{1}{1-\alpha}}(1/\alpha - 1) \quad . \end{aligned}$$

One can use the constraint to replace for Q and turn the lender's problem into a univariate problem over b'

$$\max_{b'} \left(\beta^k - \beta^f\right) b' + (\beta^f z\alpha)^{\frac{1}{1-\alpha}}(1/\alpha - 1) \quad .$$

Clearly, the lender's problem is strictly increasing in b' given that $\beta^k > \beta^f$. Thus, the lender would like to choose $b' = \infty$, which cannot be an equilibrium. Assume then that the firm's borrowing constraint binds, the optimal capital policy must satisfy $k'(z; b', Q) = b'/\theta$. The concentrated lender's problem can be written as

$$\begin{aligned} &\max_{Q,b'} W = b - Qb' + \beta^k b' \\ \text{s.t.} \quad & 0 = -b + Qb' - b'/\theta + \beta^f \left[z(b'/\theta)^\alpha - b'\right] \quad . \end{aligned}$$

Using the constraint to replace for Q, one can again turn the lender's problem into a univariate problem over b'

$$\max_{b'} \left(\beta^k - \beta^f - \frac{1}{\theta}\right) b' + \beta^f z\theta^{-\alpha}(b')^\alpha \quad .$$

The solution to this problem is

$$\begin{aligned} (b')^* &= \theta\left(\frac{\beta^f z\alpha}{1-\theta(\beta^k - \beta^f)}\right)^{\frac{1}{1-\alpha}} \quad , \\ (k')^* &= \left(\frac{\beta^f z\alpha}{1-\theta(\beta^k - \beta^f)}\right)^{\frac{1}{1-\alpha}} \quad , \\ Q^* &= \beta^f + \frac{1}{\theta}\left[1 - \frac{1-\theta(\beta^k - \beta^f)}{\alpha} + b\left(\frac{1-\theta(\beta^k - \beta^f)}{\alpha z\beta^f}\right)^{\frac{1}{1-\alpha}}\right] \quad . \end{aligned}$$

In this case, the allocations are the same as in the dispersed lending equilibrium, with the difference that the lender is willing to lend as long as $Q \leq Q^{\max}(z, b)$. The MPKs are equalized across firms.

Effectively, this solution corresponds to the bank taking over ownership of the firm and indirectly choosing investment via the binding borrowing constraint. Since the firm has no outside option (other than exit), the bank is able to extract the maximum surplus while setting the firm's value to zero. Furthermore, it holds that $Q^{\min}(z, b) \geq \beta^k \Leftrightarrow Q^* \geq \beta^k$. Thus, as long as the firm's states (z, b) are such that the firm would default in the dispersed lending case, which is the case in our benchmark, the price of debt offered by the lender Q^* will always be larger than the competitive price β^k.

Thus, in the case where the bank offers both Q and b', the allocations of b' and k' coincide with the ones of the dispersed lending case (without default), and therefore do not depend on b. The optimal price of debt Q^* is strictly increasing in b, so as to keep the firm at the participation constraint. In summary, we have that total borrowing $Q^* b'$ is increasing in b, but that total investment k' is independent from b. In contrast, our empirical analysis shows that evergreening is associated with lower interest rates, larger credit amounts, and larger investment as well. While this alternative contracting protocol is able to generate the first two observations, it does not generate the third. We therefore view the contracting protocol of our benchmark as the empirically relevant setting since it is consistent with the data in this regard.

A.4.3 Debt Forgiveness and Restructuring

Alternatively, we could allow for debt forgiveness or restructuring. A lender may prefer to write off a fraction of the legacy debt to prevent the firm from defaulting. This enables the lender to charge a higher interest rate and obtain a larger surplus on new lending. In comparison, our benchmark model implies that the lender transfers surplus to the borrower by lowering the interest rate instead of writing off debt. The solution to a model with debt forgiveness is described in this appendix. Similar to the alternative contracting protocol described above, a model with debt forgiveness predicts that debt of distressed firms should decline while interest rates remain the same for all firms, in contrast to our empirical findings, and we further show that our results are robust to excluding observations with loan charge-offs. Additionally, debt forgiveness and restructuring could entail additional costs that we do not explicitly model.

We now derive the solution to the optimal contract under the assumption that the lender can restructure or forgive part of the legacy debt b ex-post. We assume that the lender can write off a share $1-\varphi$ of legacy debt b: the amount of debt that is written off is just enough such that the firm does not default, i.e. $V(z,\varphi b)=0$, and the lender originates new debt at the risk-free price of $Q=\beta^k$.

Clearly, in the normal funding region $b \leq \bar{b}(z)$, there is no restructuring and the optimal contract is as before, with the lender setting $Q=\beta^k$. In the evergreening region $b>\bar{b}(z)$, the lender may prefer to restructure. Clearly, the lender forgives the smallest possible amount of debt that ensures that the firm is willing to operate while borrowing at $Q=\beta^k$, that is $V(z,b;Q=\beta^k)=0$. One can show that this results in

$$\varphi = \frac{1-\alpha}{\alpha b} \frac{(\beta^f \alpha z)^{\frac{1}{1-\alpha}}}{[1-\theta(\beta^k-\beta^f)]^{\frac{\alpha}{1-\alpha}}}$$

where φ is the fraction of legacy debt b that is not forgiven. Note that $\varphi \in (0,1)$ in the evergreening region, and that the bank's payoff in this region is equal to $W=\varphi b$. We have that $W>0$ as long as $\varphi>0$, thus the bank never chooses to liquidate the firm regardless of b, and the bank and the firm are strictly better off by forgiving/restructuring debt in the liquidation region than by evergreening. It can be shown that the bank's payoff from restructuring is always at least as large as that of evergreening for any (z,b).

In this case, all firms borrow at the same interest rate $Q=\beta^k$ and borrow an amount that depends on productivity z but not on the amount of legacy debt b. These predictions are at odds with the empirical evidence that we uncover in Section 3, which are also robust to excluding observations with loan charge-offs. Additionally, there may be extra costs associated with debt restructuring that we do not explicitly consider and that may make debt forgiveness a less attractive option compared to evergreening.

A.5 Idiosyncratic Risk in the Static Model

In this section, we extend the static model to include uncertainty about default at $t=0$. We assume that the firm is subject to a cost shock $c \sim G$ that is realized after the contract Q is offered. The firm may choose to default depending on the realization of this shock. We show that our main results continue to hold as long as the distribution G satisfies some general properties.

Let $V(z, b; Q)$ be defined as in (2.1). Conditional on the realization of the shock, the value of the firm is now given by

$$V_0(z, b, c; Q) = V(z, b; Q) - c \quad .$$

Note that we interpret $c \geq 0$ as an idiosyncratic cost shock, but we could relax the non-negativity assumption and treat it as a more general cash-flow or liquidity shock. Once again, we assume that the firm has limited liability and therefore chooses to default if and only if

$$V_0(z, b, c; Q) < 0 \Leftrightarrow c > V(z, b; Q) \equiv \bar{c}(z, b; Q) \quad .$$

That is, there exists a threshold level for the cost shock $\bar{c}(z, b; Q) \equiv V(z, b; Q)$ such that the firm defaults if and only if the shock realization exceeds this threshold. The ex-ante value for the firm, before the shock is realized but after the contract Q is offered, can be written as

$$\bar{V}(z, b; Q) = \mathbb{E}_c[V_0(z, b, c; Q)] = G[\bar{c}(z, b; Q)]V(z, b; Q) - \int_0^{\bar{c}(z,b;Q)} c \mathrm{d}G(c) \tag{A.3}$$

Since the investment and debt decisions (k', b') are made after the shock realization and the default decision, the firm's policy functions are still determined as in (2.1) and given by the expressions in (A.1). The expression for $V(z, b; Q)$ therefore does not change either.

We can still define $Q^{\min}(z, b)$ as in (1), with the difference that it is the threshold below which the firm defaults with probability one. If the lender offers $Q > Q^{\min}(z, b)$, the firm may still default with a positive probability that is strictly smaller than one. In other words, $Q^{\min}(z, b)$ now sets the boundaries of the region where the firm has a positive probability of survival.

Dispersed Lending. Since dispersed lenders offer lending contracts after the default decision has been made, the contract they offer is unchanged, that is, $Q = \beta^k$ conditional on survival, and 0 otherwise.

Concentrated Lending. Default is no longer deterministic and binary and the lender's problem changes to

$$W = \max_{Q \geq \beta^k} G[\bar{c}(z,b;Q)] \times \Big[b - Qb'(z;Q) + \beta^k b'(z;Q) \Big] \quad .$$

Note that instead of an indicator function equal to one in case of repayment and zero otherwise, the expression now includes a probability of survival that is equal to the probability of the firm drawing a cost shock smaller than the threshold $\bar{c}(z,b;Q)$. Similar to the benchmark case, the lender internalizes the impact of Q on the firm's threshold cost and it will take this into account when choosing the optimal contract. Since the nature of the contract that is offered by the dispersed lenders has not changed, the firm's outside option is still equal to $Q = \beta^k$, which constrains the concentrated lender's decision.

The main difference is that the bank's objective function is now concave if G satisfies some general properties. As a result, we can derive an optimal interior Q^* in the evergreening region. First, notice that Proposition 2 still holds, and there is a $Q^{\max}(z,b)$ above which the bank chooses to liquidate the firm, with the same expression as in the static model. Second, the FOC for the bank is given by

$$FOC(Q) \equiv g[\bar{c}(z,b;Q)] \Big[b + b'(z;Q)(\beta^k - Q) \Big] + G[\bar{c}(z,b;Q)] \left[\frac{\partial b'/\partial Q}{b'(z;Q)}(\beta^k - Q) - 1 \right] = 0 \tag{A.4}$$

It is useful to note that

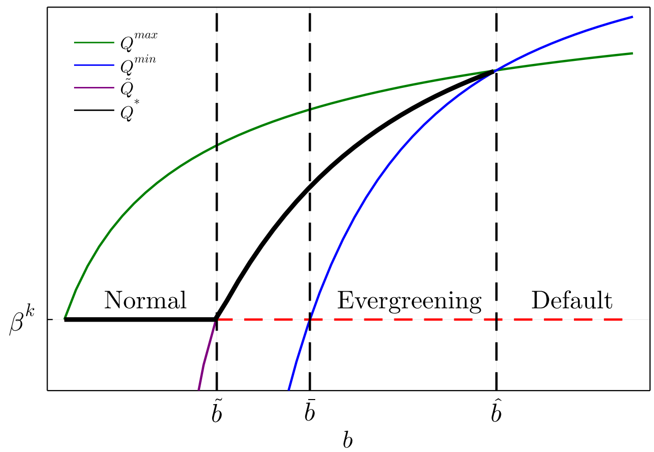
$$\frac{\partial b'/\partial Q}{b'(z;Q)} = \frac{\theta}{1-\alpha} \frac{1}{1-\theta(Q-\beta^f)} \quad .$$

Based on the second-order condition, it can be shown that the first-order condition is necessary and sufficient, if the probability density function associated with G has a non-positive slope everywhere, $g' \leq 0$, a property that is satisfied by many standard distributions such as the uniform, the Pareto, and the exponential, among others.

Let $\tilde{Q}(z,b)$ denote the solution to $FOC(Q) = 0$. It is useful to note from the FOC that we must have $\tilde{Q}(z,b) < Q^{\max}(z,b)$ since $FOC(Q^{\max}(z,b)) < 0$. We can characterize the bank's optimal policy as follows:

1. If $\tilde{Q}(z,b) \leq \max\{\beta^k, Q^{\min}(z,b)\} = \beta^k$ and $Q^{\min}(z,b) \leq Q^{\max}(z,b)$, then the

Figure A.2: Concentrated Lending Economy with Idiosyncratic Risk



Notes: Equilibrium allocation as a function of b, for a given z. The solid blue line is $Q^{\min}(z,b)$, the solid green line is $Q^{\max}(z,b)$, the solid purple line is $\tilde{Q}(z,b)$, the dashed red line is β^k, and the black line is the optimal policy Q^*.

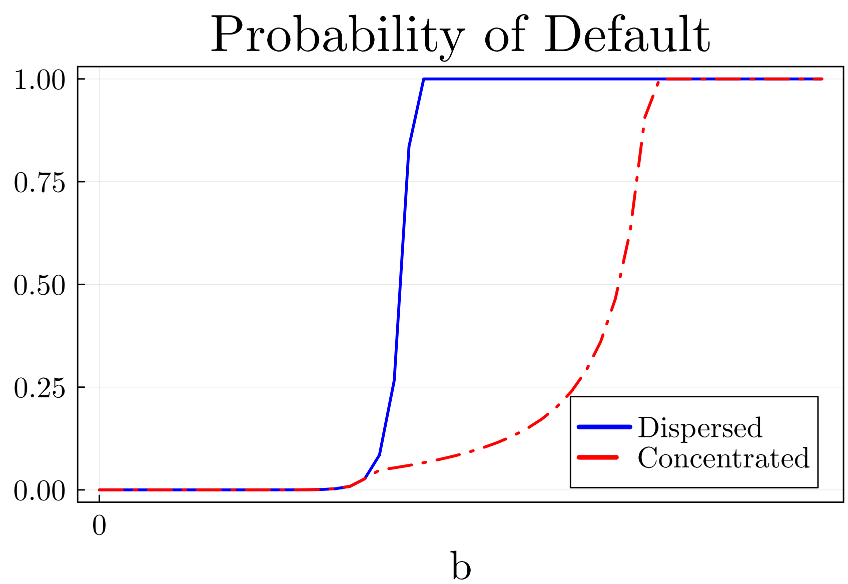
bank sets $Q^*(z,b) = \beta^k$. This corresponds to the normal funding region.

2. Finally, if $Q^{\min}(z,b) > Q^{\max}(z,b)$, the bank sets $Q^*(z,b) = 0$ and liquidates the firm.

Similar to our main model, one can further show that $Q^*(z,b)$ is weakly decreasing in z and weakly increasing in b. Figure A.2 summarizes the equilibrium in the static model with idiosyncratic firm risk based on a numerical example that assumes that G follows an exceptional distribution.Notice that in the relevant domain, we have that $\tilde{Q}(z,b) > Q^{\min}(z,b)$, implying that the evergreening region expands relative to the case with no idiosyncratic risk (i.e., evergreening begins at a lower level of debt, $\tilde{b} < \bar{b}$). Intuitively, the lender has an incentive to start subsidizing the firm for lower values of b as it trades off the increase in the expected value of repayment for a larger subsidy.

Figure A.3 shows the corresponding probabilities of default that we reference in the main text.

Figure A.3: Default Probability in an Economy with Idiosyncratic Risk



Notes: Probability of default in the static economy with idiosyncratic firm risk, as a function of b for a given z. The solid line corresponds to the dispersed lending case, while the dashed line corresponds to the concentrated lending economy.

B Data

Table B.1: Compustat Variable Definitions.

Variable Name	**Description**	**Compustat Name**
Total Assets	Total firm assets	atq
Employer Identification Number	Used to match to TIN in Y14	ein
Total Liabilities	Total firm liabilities	ltq
Net Income	Firm net income (converted to 12-month trailing series)	niq
Total Debt	Debt in current liabilities + long-term debt	dlcq + dlttq
Sales	Total firm sales	saleq
Fixed Assets	Net property, plant, and equipment	ppentq
Receivables	Receivables	rectq
Inventories	Inventories	invtq
Cash	Cash & Marketable securities	cheq

Notes: All data obtained from the Wharton Research Data Services. Nominal series deflated using the consumer price index for all items taken from St. Louis Fed's FRED database.

Table B.2: Variables from Y-9C filings.

Variable Code	**Variable Label**
BHCK 2170	Total Assets
BHCK 2948	Total Liabilities
BHCK 4340	Net Income
BHCK 3197	Earning assets that reprice or mature within one year
BHCK 3296	Interest-bearing deposit liabilities that reprice or mature within one year
BHCK 3298	Long-term debt that reprices within one year
BHCK 3408	Variable-rate preferred stock
BHCK 3409	Long-term debt that matures within one year
BHDM 6631	Domestic offices: noninterest-bearing deposits
BHDM 6636	Domestic offices: interest-bearing deposits
BHFN 6631	Foreign offices: noninterest-bearing deposits
BHFN 6636	Foreign offices: interest-bearing deposits
BHCA 7206	Tier 1 Capital Ratio
BHCK B529	Loans and Leases held for investment
BHCK 5369	Loans and Leases held for sale

Notes: The table lists variables that are collected from the Consolidated Financial Statements or FR Y-9C filings for Bank-Holding Companies from the Board of Governors' National Information Center database. The one-year income gap is defined as $(\text{BHCK}\,3197 - (\text{BHCK}\,3296 + \text{BHCK}\,3298 + \text{BHCK}\,3408 + \text{BHCK}\,3409))\,/\text{BHCK}\,2170$. Total deposits are given by $(\text{BHDM}\,6631 + \text{BHDM}\,6636 + \text{BHFN}\,6631 + \text{BHFN}\,6636)$. Nominal series are deflated using the consumer price index for all items taken from St. Louis Fed's FRED database.

Table B.3: FR Y-14 Variable Definitions.

Variable Name	Description / Use	Field No.
Zip code	Zip code of headquarters	7
Industry	Derived 2-Digit NAICS Code	8
TIN	Taxpayer Identification Number	11
Internal Credit Facility ID	Used together with BHC and previous facility ID to construct loan histories	15
Previous Internal Credit Facility ID	Used together with BHC and facility ID to construct loan histories	16
Term Loan	Loan facility type reported as Term Loan, includes Term Loan A-C, Bridge Loans, Asset-Based, and Debtor in Possession.	20
Credit Line	Loan facility type reported as revolving or non-revolving line of credit, standby letter of credit, fronting exposure, or commitment to commit.	20
Purpose	Credit facility purpose	22
Used Credit	Utilized credit exposure	25
Line Reported on Y-9C	Line number reported in HC-C schedule of FR Y-9C	26
Cumulative Charge-offs	Cumulative Charge-offs	28
Participation Flag	Used to determine whether a loan is syndicated	34
Variable Rate	Interest rate variability reported as "Floating" or "Mixed"	37
Interest Rate	Current interest rate	38
Date Financials	Financial statement date used to match firm financials to Y-14 date	52
Net Sales Current	Firm sales over trailing 12-month period	54
Interest Expenses	Used in calculating average interest rate on all debt	58
Net Income	Current net income for trailing 12-months used to construct return on assets	59, 60
Cash	Cash & Markatable Securities	61
Tangible Assets	Tangible assets	68
Total Assets	Total assets, current year and prior year	70
Short Term Debt	Used in calculating total debt	74
Long Term Debt	Used in calculating total debt	78
Probability of Default	Probability of default for firms (corresponds to internal risk rating for non-advanced BHCs)	88
Syndicated Loan	Syndicated loan flag	100

Notes: Nominal series are converted into real series using the consumer price index for all items taken from St. Louis Fed's FRED database. The corresponding "Field No." can be found in the data dictionary (Schedule H.1, pp. 162-217): https://www.federalreserve.gov/reportforms/forms/FR_Y-14Q20200331_i.pdf

B.1 Sample Restrictions and Filtering Steps

1. We constrain the sample to loan facilities with line reported on the HC-C schedule in the FR Y9-C filings as commercial and industrial loans, "other" loans, "other" leases, and owner-occupied commercial real estate (corresponding to Field No. 26 in the H.1 schedule of the Y14 to be equal to 4, 8, 9, or 10; see Table B.3). In addition, we drop all observations with NAICS codes 52 and 53 (loans to financial firms and real estate firms).

2. Observations with negative or zero values for committed exposure, negative values for utilized exposure, with committed exposure less than utilized exposure are excluded, and gaps in their loan histories.

3. When aggregating loans at the firm-level, we exclude observations for which the firm identifier "TIN" is missing. To preserve some of these missing values, we fill in missing TINs from a history where the non-missing TIN observations are all the same over a unique facility ID.

4. When using information on firms' financials in the analysis, we apply a set of filters to ensure that the reported information is sensible. We exclude observations (i) if total assets, total liabilities, short-term debt, long-term debt, cash assets, tangible assets, or interest expenses are negative, (ii) if tangible assets, cash assets, or total liabilities are greater than total assets, and (iii) if total debt (short term + long term) is greater than total liabilities.

5. When using the interest rate on loans in our calculations, we exclude observations with interest rates below 0.5 or above 50 percent to minimize the influence of data entry errors.

B.2 Bank Capital

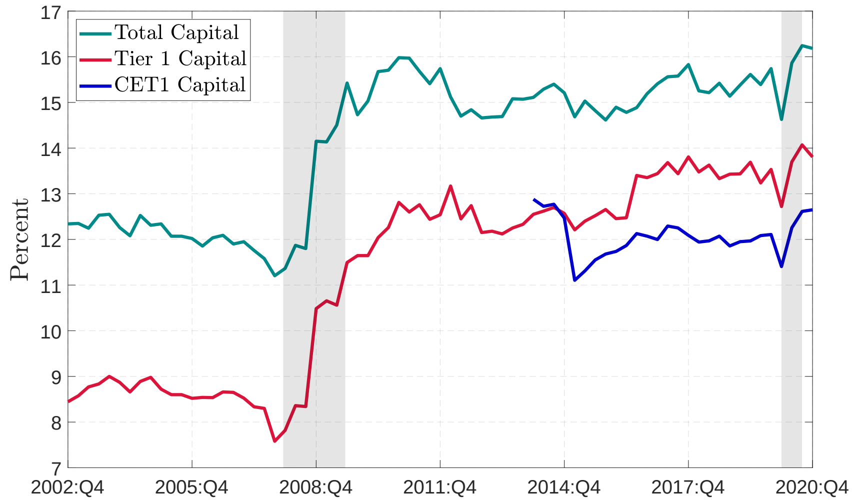


Figure B.1: Bank Capital Ratios.

Notes: For each date, the figure shows the median of the CET1, Tier 1, and total capital ratios across the Y14-banks. Gray bars denote NBER recessions.

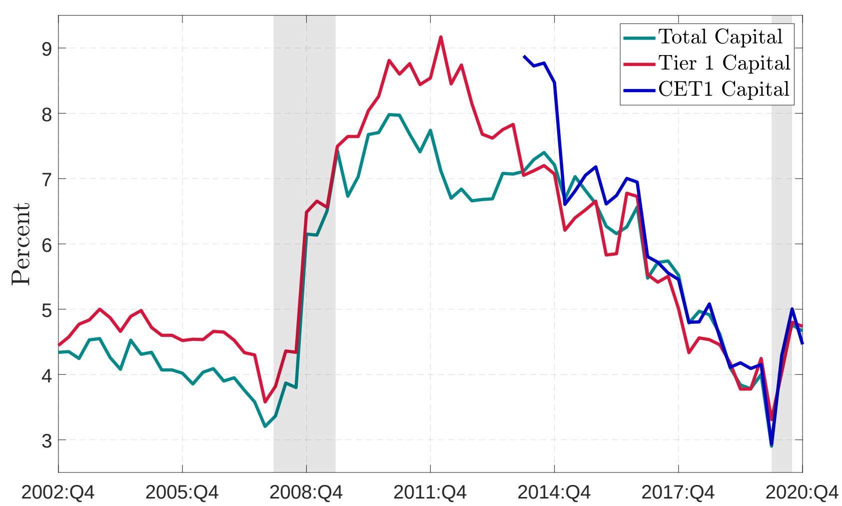


Figure B.2: Bank Capital Buffers.

Notes: For each date, the figure shows the median of the CET1, Tier 1, and total capital buffer across the Y14-banks. Capital buffers are defined as the difference between capital ratios and requirements. Gray bars denote NBER recessions.

C Identifying Credit Supply Effects: Extensions & Robustness

In this section, we explore extensions and consider the robustness of our empirical findings.

First, we investigate whether our findings can be explained by an alternative channel, as opposed to the mechanism working through debt exposures and firm financial distress. We are particularly interested in testing whether alternative theories of evergreening or zombie lending based on bank capital positions—such as gambling for resurrection and risk-shifting—drive our results. Various bank controls, including bank capital buffers, and the bank-time fixed effects that are part of the regressions reported in Table 3.1 already account for a number of alternative mechanisms. We further include interaction terms between bank or firm controls and Debt-Share$_{i,j,t}$ or Distress$_{i,t}$ into our baseline setup. The estimation results are shown in Appendix Table C.3. Even with the various interaction terms, the estimates remain close to the ones from our benchmark specification, providing evidence in favor of our theory over alternative ones.

Second, we exclude observations with any loan charge-offs to address the possibility that our findings could be affected by an alternative mechanism of debt forgiveness or restructuring. There are relatively few such observations, leading to regression results that are nearly identical to our baseline estimates as shown in Appendix Table C.4.

Third, we test the sensitivity of our findings to the chosen cutoff value for $\overline{PD}_{i,t}$ that defines distressed firms. Appendix Table C.5 reports the results for three alternatives. As opposed to the top 10 percent, we widen and narrow the definition, considering the top 5 or top 15 percent of the unconditional distribution for $\overline{PD}_{i,t}$ as cutoff values instead. Moreover, our theory shows that evergreening should not occur for the part of the firm population for which default is unavoidable (see Figures 2.1 and A.3). Starting from our baseline, we therefore also consider a third alternative, defining a firm as nondistressed if its PD lies above the 95th percentile of $\overline{PD}_{i,t}$. For all these alternatives, the estimated coefficients are close to the ones from our benchmark specification. However, we also note that our key results start to vanish if we broaden the definition for distressed firms further. For example, if we consider the 75th percentile of the unconditional distribution of $\overline{PD}_{i,t}$ as a cutoff value for $\text{Distress}_{i,t}$, which is equivalent to a PD of 1.9 percent, β_2 in the credit regressions reduces in magnitude and is not statistically different from zero at standard confidence levels. Thus, our findings are not sensitive to the exact PD value that defines a distressed firm, as long as we capture a part of the firm population that has a reasonable chance of default.

Fourth, while we consider the reported PDs as valid indicators of firm financial distress, banks may misreport such statistics in practice. As documented in Plosser and Santos (2018) and Behn, Haselmann and Vig (2022), low-capitalized banks systematically underreport their credit risk exposure to firms, which may distort our definition for distressed firms. While the bank-specific controls in our baseline regression partly address such a concern, we further exclude banks with total capital buffers below the 5th, 10th, and 25th percentiles of the overall distribution of bank capital buffers. For each respective sample, we also recompute the average firm-specific $\overline{PD}_{i,t}$. The estimation results are reported in Appendix Table C.6, which remain similar to the ones from our baseline specification, even for these substantially reduced samples. These results also illustrate that our findings do not hinge on bank health, consistent with our theory.

Fifth, a related concern is that banks may disagree about firms' likelihood of default and the disagreement may be correlated with $\text{Debt-Share}_{i,j,t}$, leading us to interpret different views about default risk as different debt share exposures. To address such a concern, we follow two approaches. First, we define the variable $\text{PD-Gap}_{i,j,t} = PD_{i,j,t} - \overline{PD}_{i,t}$, which measures the difference between bank j's reported PD for firm i and the average for that firm across all banks. A first indication that bank disagreement does not affect our results is that $\text{PD-Gap}_{i,j,t}$ and $\text{Debt-Share}_{i,j,t}$ are nearly uncorrelated for our baseline estimation sample, with a correlation coefficient of -0.01. To further investigate the role of disagreement, we omit observations with absolute $\text{PD-Gap}_{i,j,t}$ values above 5 percentage points, corresponding to excluding approximately the 25 percent of the largest and smallest observations for distressed firms. Second, we directly control for $PD_{i,j,t}$ in our main regression setup. The estimation results for these two robustness checks—which remain similar to our original ones—are shown in Appendix Table C.7.

Sixth, loan contract terms such as maturity, collateral, and whether a loan is syndicated may correlate with the loan size. To ensure that we compare loans with similar contract terms, we extend the firm-time fixed effects with such characteristics. Appendix Table C.8 shows the updated estimation results, which are again similar to our baseline estimates.

Seventh, our regression setup differs from the one in Khwaja and Mian (2008) who run a change-on-change regression, while we regress changes in bank credit provision on the intensity of the bank-firm relation and a distress indicator. To our align our specification with their setup, we consider instead a distress indicator that equals one for firms that transition into financial distress, that is, their PD rises from one period to the next above the cutoff value. The estimation results are reported in Appendix Table C.9. While the findings are much the same, we prefer our original specification since (i) it covers a larger sample of distressed firms and (ii) lenders may not immediately respond to a firm becoming distressed due to loan rigidities inherent with long-term contracts and because of learning frictions.

And last, we include credit lines into the estimation sample. The new results are shown in Appendix Table C.10. For the credit regressions, our findings remain. However, for the interest rate regressions, the coefficients associated with the interaction term Debt-Share$_{i,j,t}$ × Distress$_{i,t}$ are not statistically different from zero at standard confidence levels, indicating that the credit movements may reflect demand shifts rather than supply effects.

Table C.1: PD Distribution and Comparison with Zombie Measures.

Measure	Observations	Correlation Distress	Indicator Value	PD Distribution P10	P50	P75	P90	P95	P99
PD Baseline	51,869	0.54	–	.17	.82	1.91	3.89	7.75	35.24
CHK	189,388	-0.04	1	.15	.66	1.56	3.73	6.57	25.16
			0	.18	.97	2.08	5.07	10.01	35.42
SST	200,156	0.22	1	.31	1.62	3.98	10.22	19.88	100
			0	.17	.73	1.6	3.5	5.9	20
FMP	79,119	0.20	1	.23	1.85	8.07	22.94	61.35	100
			0	.16	.67	1.53	3.7	6.65	23.54
Model	245,341	0.14	1	.43	2.8	7.16	19.73	30	100
			0	.17	.76	1.77	3.73	6.92	22.7

Notes: "PD Baseline" shows the unconditional distribution of $\overline{PD}_{i,t}$ for term loans used in our analysis. The remaining measures define zombie firms according to various characteristics: "CHK"=Caballero, Hoshi and Kashyap (2008), "SST"=Schivardi, Sette and Tabellini (2022), "FMP"=Favara, Minoiu and Perez-Orive (2022), Model=model equivalent based on high leverage and low profitability (see text for descriptions). "Correlation Distress" is the correlation coefficient of the various measures with the distress indicator Distress$_{i,t}$.

Table C.2: Summary Statistics.

Variable	Obs.	Mean	Std.	P5	Median	P95
Firm-Specific						
Debt-Share	8,647	0.08	0.11	0.00	0.03	0.30
Distress	8,647	0.06	0.24	0.00	0.00	1.00
Bank Controls						
ROA	519	0.01	0.00	0.00	0.01	0.01
Income Gap	519	0.37	0.11	0.16	0.37	0.51
Leverage	519	0.88	0.02	0.85	0.88	0.92
Ln(Total Assets)	519	19.4	1.10	18.0	19.1	21.5
Deposit Share	519	0.63	0.18	0.18	0.70	0.82
Loan Share	519	0.49	0.20	0.11	0.54	0.71
Tier 1 Capital Buffer	519	6.19	3.55	2.61	5.45	11.1
Unused Credit/Assets	519	0.11	0.06	0.02	0.10	0.21

Notes: The table reports summary statistics for the variables used in the baseline regressions (3.1) in Table 3.1.

Table C.3: Credit Supply to Distressed Firms - Interaction Terms.

	Δ Credit			Δ Interest Rate		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt-Share	-22.03**	-26.89**	-39.83	0.17***	0.21**	0.24*
	(8.25)	(11.82)	(27.82)	(0.05)	(0.09)	(0.13)
Debt-Share × Distress	37.03***	40.07***	38.41***	-0.66*	-0.90***	-0.70**
	(11.54)	(9.29)	(11.94)	(0.33)	(0.29)	(0.30)
Interaction Terms						
Bank Controls × Distress	✓			✓		
Bank Controls × Debt-Share		✓			✓	
Firm Controls × Debt-Share			✓			✓
Bank Controls	✓	✓	✓	✓	✓	✓
Firm × Time FE	✓	✓	✓	✓	✓	✓
R-squared	0.58	0.59	0.59	0.74	0.74	0.76
Observations	8,647	8,647	8,045	8,407	8,407	7,819
w/ Distress = 1	539	539	464	528	528	453
Number of Firms	887	887	834	867	867	815
Number of Banks	36	36	36	36	36	36

Notes: Estimation results for regression (3.1) multiplied by 100. All specifications include firm-time fixed effects and various bank controls: bank size (natural log of assets), return on assets (net income/assets), deposit share (total deposits/assets), loan share (loans/assets), leverage (liabilities/assets), Tier 1 capital buffer (ratio minus requirement), banks' income gap, and the ratio of unused credit lines to assets. Columns (i) and (iv) include interactions terms between those bank controls and the firm distress indicator, columns (ii) and (v) between the bank controls and the debt share, and columns (iii) and (vi) between various firm controls and the debt share. The firm controls include cash holdings, net income, liabilities, tangible assets (all scaled by total assets), and firm size (natural log of total assets). Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2014:Q4 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table C.4: Credit Supply to Distressed Firms - Loan Charge-offs.

	Δ Credit			Δ Interest Rate		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt-Share	-21.99**	-17.52**	-22.57***	0.18***	0.11	0.13*
	(8.25)	(8.58)	(7.85)	(0.05)	(0.07)	(0.06)
Debt-Share × Distress	44.16***	36.85***	45.78***	-0.89**	-0.71**	-0.66**
	(9.55)	(10.46)	(12.88)	(0.33)	(0.34)	(0.32)
Fixed Effects						
Firm × Time	✓		✓	✓		✓
Firm × Time × Pur.		✓			✓	
Bank × Time			✓			✓
Bank Controls	✓	✓		✓	✓	
R-squared	0.58	0.6	0.63	0.75	0.74	0.8
Observations	8,629	5,717	8,554	8,389	5,549	8,316
w/ Distress = 1	523	387	517	512	376	506
Number of Firms	885	641	882	865	620	862
Number of Banks	36	34	34	36	34	34

Notes: Estimation results for regression (3.1) multiplied by 100. All specifications exclude observations with any historic loan charge-offs until time t or at time $t+2$. All regressions include firm-time fixed effects that additionally vary by the loan purpose in columns (ii) and (v). Columns (iii) and (vi) include bank-time fixed effects and the remaining columns include various bank controls: bank size (natural log of assets), return on assets (net income/assets), deposit share (total deposits/assets), loan share (loans/assets), leverage (liabilities/assets), Tier 1 capital buffer (ratio minus requirement), banks' income gap, and the ratio of unused credit lines to assets. Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2014:Q4 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table C.5: Credit Supply to Distressed Firms - Distress Cutoffs.

	Δ Credit			Δ Interest Rate		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt-Share	-20.17**	-21.66**	-21.20**	0.15**	0.17***	0.16***
	(8.19)	(8.19)	(8.16)	(0.06)	(0.05)	(0.05)
Debt-Share $\times$ Distress	39.99***	33.14**	46.56***	-1.23*	-0.64**	-0.76*
	(13.40)	(13.23)	(10.97)	(0.65)	(0.31)	(0.38)
Distress Cutoffs						
$\overline{PD} \geq \kappa_{95}$	✓			✓		
$\overline{PD} \geq \kappa_{85}$		✓			✓	
$\kappa_{95} > \overline{PD} \geq \kappa_{90}$			✓			✓
Firm $\times$ Time FE	✓	✓	✓	✓	✓	✓
Bank Controls	✓	✓	✓	✓	✓	✓
R-squared	0.58	0.58	0.58	0.74	0.74	0.74
Observations	8,647	8,647	8,647	8,407	8,407	8,407
w/ Distress = 1	304	711	235	296	697	232
Number of Firms	887	887	887	867	867	867
Number of Banks	36	36	36	36	36	36

Notes: Estimation results for regression (3.1) multiplied by 100. Columns (i) and (iv) alter the distress cutoff to the 95th percentile (7.75%) of the unconditional distribution of $\overline{PD}_{i,t}$, columns (ii) and (v) to the 85th percentile (2.88%), and columns (iii) and (vi) define distressed firms to have PDs that lie between the 90th (3.89%) and 95th (7.75%) percentiles. All specifications include firm-time fixed effects and various bank controls: bank size (natural log of assets), return on assets (net income/assets), deposit share (total deposits/assets), loan share (loans/assets), leverage (liabilities/assets), Tier 1 capital buffer (ratio minus requirement), banks' income gap, and the ratio of unused credit lines to assets. Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2014:Q4 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table C.6: Credit Supply to Distressed Firms - Bank Capital.

		Δ Credit			Δ Interest Rate	
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt-Share	-21.80**	-24.11***	-29.68***	0.16***	0.19***	0.22***
	(8.04)	(8.56)	(10.11)	(0.06)	(0.06)	(0.08)
Debt-Share × Distress	41.29***	44.87***	52.26***	-0.91**	-0.87*	-1.05*
	(9.39)	(13.54)	(16.44)	(0.35)	(0.43)	(0.55)
Bank Capital Cutoffs						
Cap-Buffer>p5	✓			✓		
Cap-Buffer>p10		✓			✓	
Cap-Buffer>p25			✓			✓
Firm × Time FE	✓	✓	✓	✓	✓	✓
Bank Controls	✓	✓	✓	✓	✓	✓
R-squared	0.57	0.57	0.59	0.72	0.72	0.71
Observations	7,845	6,978	5,614	7,624	6,768	5,443
w/ Distress = 1	473	389	319	462	378	310
Number of Firms	836	784	690	817	764	673
Number of Banks	36	36	35	36	36	34

Notes: Estimation results for regression (3.1) multiplied by 100. Columns (i) and (iv) restrict the sample to banks with total capital buffers (ratio - requirement) above the 5th percentile across all banks (2.72%), columns (ii) and (v) above the 10th percentile (3.31%), and columns (iii) and (vi) above the 25th percentile (4.42%). All specifications include firm-time fixed effects and various bank controls: bank size (natural log of assets), return on assets (net income/assets), deposit share (total deposits/assets), loan share (loans/assets), leverage (liabilities/assets), Tier 1 capital buffer (ratio minus requirement), banks' income gap, and the ratio of unused credit lines to assets. Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2014:Q4 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table C.7: Credit Supply to Distressed Firms - PD Differences.

	Δ Credit			Δ Interest Rate		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt-Share	-21.88**	-9.72	-9.52	0.18***	0.15***	0.16***
	(8.24)	(5.92)	(5.88)	(0.05)	(0.05)	(0.06)
Debt-Share × Distress	45.60***	23.27**	23.30**	-0.93***	-1.05**	-1.17***
	(9.49)	(10.51)	(9.96)	(0.33)	(0.41)	(0.35)
PD			-0.14			-0.00
			(0.09)			(0.00)
∣PD-Gap∣<5		✓			✓	
Firm × Time FE	✓	✓	✓	✓	✓	✓
Bank Controls	✓	✓	✓	✓	✓	✓
R-squared	0.58	0.53	0.53	0.74	0.75	0.74
Observations	8,647	7,232	7,498	8,407	7,136	7,402
w/ Distress = 1	539	232	488	528	230	486
Number of Firms	887	751	770	867	744	763
Number of Banks	36	30	30	36	30	30

Notes: Estimation results for regression (3.1) multiplied by 100. Columns (ii) and (v) restrict the sample to observations with $|\text{PD-Gap}_{i,j,t}| = |PD_{i,j,t} - \overline{PD}_{i,t}| < 5$. Columns (iii) and (vi) include $PD_{i,j,t}$ as a control. All specifications include firm-time fixed effects and various bank controls: bank size (natural log of assets), return on assets (net income/assets), deposit share (total deposits/assets), loan share (loans/assets), leverage (liabilities/assets), banks' income gap, and the ratio of unused credit lines to assets. Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2014:Q4 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table C.8: Credit Supply to Distressed Firms - Alternative Fixed Effects.

	Δ Credit			Δ Interest Rate		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt-Share	-10.78*	-10.89*	-9.54*	0.19***	0.17***	0.17***
	(6.04)	(5.58)	(5.03)	(0.05)	(0.05)	(0.05)
Debt-Share × Distress	32.79***	32.99***	35.97***	-0.75**	-0.84**	-0.66*
	(9.04)	(7.59)	(10.51)	(0.32)	(0.34)	(0.37)
Fixed Effects						
Firm × Time × Maturity	✓			✓		
Firm × Time × Securitized		✓			✓	
Firm × Time × Syndicated			✓			✓
Bank Controls	✓	✓	✓	✓	✓	✓
R-squared	0.52	0.51	0.53	0.76	0.76	0.72
Observations	8,319	7,963	6,706	8,175	7,835	6,563
w/ Distress = 1	505	515	393	494	506	379
Number of Firms	854	809	750	845	802	738
Number of Banks	36	36	35	36	36	35

Notes: Estimation results for regression (3.1) multiplied by 100. All specifications include firm-time fixed effects that additionally vary by a loan's maturity in columns (i) and (iv) (one quarter or less, one year or less, or more than one year), whether the loan is securitized in columns (ii) and (v), or whether the loan is syndicated in columns (iii) and (vi). All specifications include various bank controls: bank size (natural log of assets), return on assets (net income/assets), deposit share (total deposits/assets), loan share (loans/assets), leverage (liabilities/assets), Tier 1 capital buffer (ratio minus requirement), banks' income gap, and the ratio of unused credit lines to assets. Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2014:Q4 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table C.9: Credit Supply to Distressed Firms - Transitions.

	Δ Credit		Δ Interest Rate	
	(i)	(ii)	(iii)	(iv)
Debt-Share	-21.88**	-22.26**	0.18***	0.17***
	(8.24)	(8.26)	(0.05)	(0.05)
Debt-Share × Distress	45.60***		-0.93***	
	(9.49)		(0.33)	
Debt-Share ×Δ Distress		35.55**		-1.02*
		(14.92)		(0.56)
Firm × Time FE	✓	✓	✓	✓
Bank Controls	✓	✓	✓	✓
R-squared	0.58	0.58	0.74	0.75
Observations	8,647	8,182	8,407	7,952
w/ Distress = 1	539	74	528	73
Number of Firms	887	845	867	827
Number of Banks	36	36	36	36

Notes: Estimation results for regression (3.1) multiplied by 100. Columns (ii) and (iv) replace the distress indicator $\text{Distress}_{i,t}$ with an indicator $\Delta\text{Distress}_{i,t}$ that equals one if $\text{Distress}_{i,t}$ switches from zero to one between $t-1$ and t and is zero if $\text{Distress}_{i,t}$ is equal to zero. All specifications include firm-time fixed effects and various bank controls: bank size (natural log of assets), return on assets (net income/assets), deposit share (total deposits/assets), loan share (loans/assets), leverage (liabilities/assets), banks' income gap, and the ratio of unused credit lines to assets. Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2014:Q4 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table C.10: Credit Supply to Distressed Firms - Credit Lines.

	Δ Credit			Δ Interest Rate		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt-Share	-61.52***	-62.65***	-61.56***	0.19**	0.14***	0.18**
	(6.75)	(8.61)	(7.01)	(0.08)	(0.05)	(0.08)
Debt-Share × Distress	22.26*	37.97**	20.66*	-0.18	-0.42	-0.20
	(11.32)	(14.77)	(11.31)	(0.33)	(0.29)	(0.30)
Fixed Effects						
Firm × Time × CL	✓		✓	✓		✓
Firm × Time × CL × Pur.		✓			✓	
Bank × Time			✓			✓
Bank Controls	✓	✓		✓	✓	
R-squared	0.84	0.88	0.84	0.64	0.7	0.67
Observations	89,172	61,886	89,167	75,773	52,388	75,769
w/ Distress = 1	10,230	6,935	10,230	8,675	5,875	8,675
Number of Firms	4,126	3,110	4,126	3,692	2,765	3,692
Number of Banks	36	36	36	36	36	36

Notes: Estimation results for regression (3.1) multiplied by 100. All specifications include firm-time fixed effects that additionally vary by whether the loan is a credit line or a term loan and by the loan purpose in columns (ii) and (v). Columns (iii) and (vi) include bank-time fixed effects and the remaining columns include various bank controls: bank size (natural log of assets), return on assets (net income/assets), deposit share (total deposits/assets), loan share (loans/assets), leverage (liabilities/assets), Tier 1 capital buffer (ratio minus requirement), banks' income gap, and the ratio of unused credit lines to assets. Standard errors in parentheses are two-way clustered by bank and firm. Sample: 2014:Q4 - 2019:Q4. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

D Dynamic model

D.1 Proofs

Proof of Proposition 4. Given the fixed supply of labor N, a given stock of capital K, and a distribution of firms $\lambda(s)$ with mass M, we can write the planner's problem as

$$\max_{k,n} \int zk^{\alpha}n^{\nu}\mathrm{d}\lambda(s)$$
$$\text{s.t.} \quad \int k\mathrm{d}\lambda(s) \leq K \quad ,$$
$$\int n\mathrm{d}\lambda(s) \leq N \quad .$$

It is straightforward to show, after some algebra, that the solution to this problem is given by

$$k = \frac{z^{\frac{1}{1-\alpha-\nu}}}{\int z^{\frac{1}{1-\alpha-\nu}}\mathrm{d}\lambda(s)}K \quad , \quad n = \frac{z^{\frac{1}{1-\alpha-\nu}}}{\int z^{\frac{1}{1-\alpha-\nu}}\mathrm{d}\lambda(s)}N \quad .$$

Notice that the planner equates the MPK across all firms. Computing aggregate TFP then gives us

$$TFP^* = \frac{\int z\left[\frac{z^{\frac{1}{1-\alpha-\nu}}}{\int z^{\frac{1}{1-\alpha-\nu}}\mathrm{d}\lambda(s)}K\right]^{\alpha}\left[\frac{z^{\frac{1}{1-\alpha-\nu}}}{\int z^{\frac{1}{1-\alpha-\nu}}\mathrm{d}\lambda(s)}N\right]^{\nu}\mathrm{d}\lambda(s)}{K^{\alpha}N^{\nu}} = \left[\int z^{\frac{1}{1-\alpha-\nu}}\mathrm{d}\lambda(s)\right]^{1-\nu-\alpha} \quad .$$

This allows us to write output in the planner's economy as in the proposition:

$$Y^* = TFP^*K^{\alpha}N^{\eta} = M^{1-\alpha-\eta}\mathbb{E}[z^{\frac{1}{1-\alpha-\nu}}]^{1-\nu-\alpha}K^{\alpha}N^{\eta} \quad .$$

D.2 Zombie Firm Classifications

We describe in more detail how our measure of subsidized firms correlates with different types of zombie classifications that are used in the literature. We focus on the following classification measures:

1. Favara, Minoiu and Perez-Orive (2022) (FMP): (i) Leverage above median; (ii) Interest-coverage ratio (ICR) below 1; (iii) Negative average sales growth

over the previous 3 years.

2. McGowan, Andrews and Millot (2018) (MAM): (i) ICR below 1 for 3 consecutive years; (ii) At least 10 years old.

3. Schivardi, Sette and Tabellini (2022) (SST): (i) Return on assets below the risk-free rate; (ii) Leverage above 40%.

4. Banerjee and Hofmann (2022) (BH): (i) ICR below 1 for 2 consecutive years; (ii) Tobin's Q below median.

5. Caballero, Hoshi and Kashyap (2008) (CHK): (i) Interest rate below the risk-free rate.

Table D.1: Zombie classification criteria in the CLE.

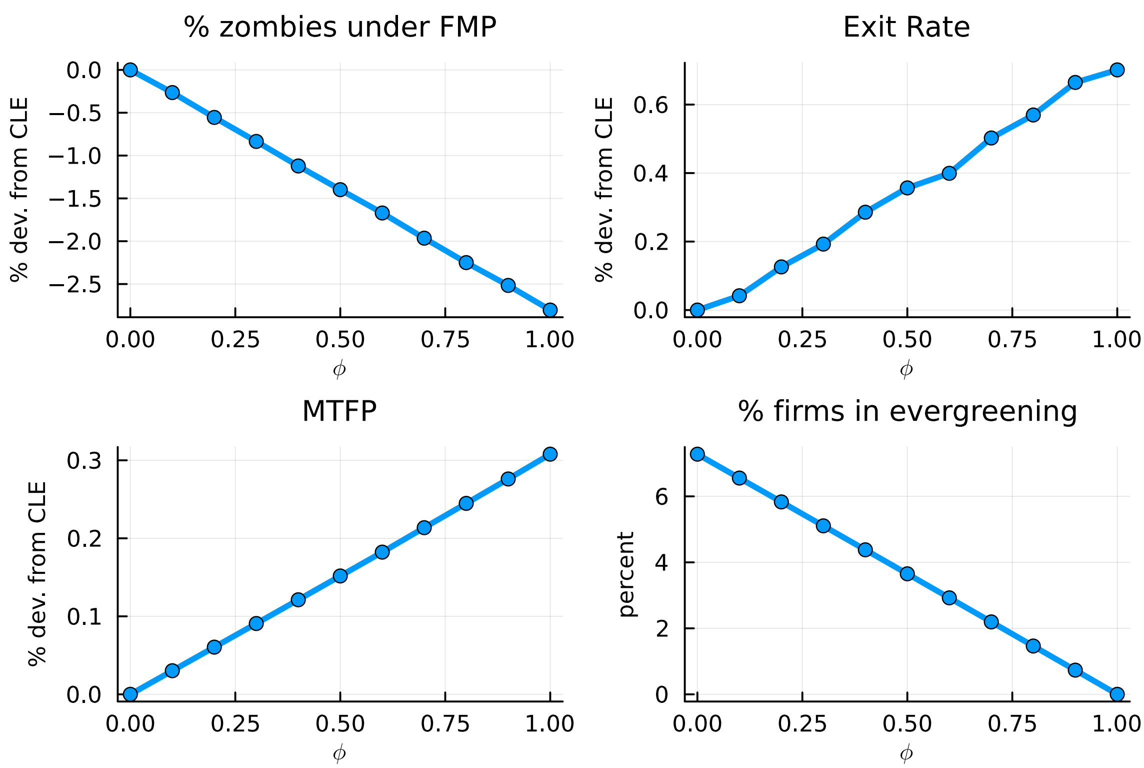
	% of Zombies	False Positives, %	True Positives, %	Balanced Accuracy, %
FMP	5.67	4.05	28.35	62.15
MAM	6.53	6.58	5.70	49.56
SST	24.20	19.94	83.97	82.01
BH	2.59	2.34	6.19	51.93
CHK	0.01	0.00	0.13	50.07

Table D.1 reports the results from applying each of these definitions to the CLE, along with different measures of diagnostic ability relative to our definition of subsidized firms. We consider the false positive rate (FPR), which is equal to the ratio of false positives to positives, the true positive rate (TPR), equal to the ratio of true positives to positives, and the balanced accuracy measure, which is the average between the TPR and true negative rates (TNR). Within our model, the SST and FMP measures are most successful in achieving a high TPR relative to FPR.[25] These are also the two measures that do the best in terms of the balanced accuracy measure, which also takes into account a classification measure's ability to correctly identify non-zombies (the TNR).

[25]In receiver operating characteristic (ROC) analysis, a binary classifier is considered to be perfect if it attains a FPR of 0 and a TPR of 1.

D.3 Firm Level Effects: Data vs. Model

Figure D.1: Aggregate moments as a function of ϕ.



Notes: This figure plots the share of firms classified as zombies according to the definition of Favara, Minoiu and Perez-Orive (2022), the exit rate, Measured TFP, and the share of firms in the evergreening region as a function of ϕ (the share of entrants that are assigned dispersed lenders).

Table D.2: Firm level Effects based on Model Simulation.

	Δ Total Debt	Investment
HHI	0.383	0.312
HHI$\times$Distress	2.651	5.094
Distress	-26.588	-81.848

Notes: Estimation results for regression (3.2) on data simulated from the model, multiplied by 100. The table reports average regressions coefficients from 500 simulated panels. To generate each panel, we simulate 20,000 distinct firms for 100 periods, starting from the stationary distribution. We consider only the last 5 periods of the simulation to generate a sample length that is similar to that in the data. A fraction $\phi = 0.09$ of entrants is assigned dispersed lenders permanently and $HHI_{i,t} = 0$, while a complementary fraction is assigned concentrated lenders and $HHI_{i,t} = 1$. $Distress_{i,t}$ is a binary variable equal to 1 if a concentrated borrowing firm is subsidized, or if the concentrated borrowing pricing functions would have implied a subsidy given each firm's states. Additional controls include size (measured as the log of capital) and leverage (debt over capital), as well as the interaction of these two variables (demeaned) with $Distress_{i,t}$.